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ANALYSIS OF A PSEUDO-NOISE ADDRESSING SYSTEM FOR MULTIPLE ACCESS COMMUNICATION WITH APPLICATIONS OF ERROR-CORRECTING CODES

by

Oscar M. Bull

United States
Naval Postgraduate School

THESIS

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Oscar M. Bull

June 1970

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Analysis of a Pseudo-Noise Addressing System for Multiple Access Communication with Applications of Error-Correcting Codes

by

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Submitted in partial fulfillment of the
requirements for the degrees

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

and

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ABSTRACT

This thesis describes several modulation schemes to be used in a random access discrete address (RADA) system. This RADA system uses only one common frequency as carrier, and no synchronization of the net is supposed. Recognition of the transmissions between pairs of talkers involves the use of pseudo-random sequences as individual addresses of each subscriber.

Detection of a particular pseudo-random sequence, contaminated by the noise formed by the simultaneous transmissions of other sequences, is accomplished by correlation methods.

A comparative study is presented for the transmission of the information by the RADA system. The following methods are described and analyzed: an M-ary system, where the information is represented by cyclic shifts of the pseudo-random sequence used as the recipient's address, and pulse code modulation (PCM) where the information is represented by a "1" or "0".

Finally, the effect of encoding the information digits via error-correcting codes is investigated. Here, use of cyclic block codes and convolutional codes are presented. It is concluded that the use of convolutional codes would improve the performance of RADA systems.

TABLE OF CONTENTS

I. INTRODUCTION -- 13

II. DESCRIPTION OF THE MODEL FOR THE SYSTEM --------- 17

III. PROPERTIES OF THE PSEUDO-RANDOM SEQUENCES ------- 20

- A. GENERAL -- 20
- B. AUTOCORRELATION FUNCTION -------------------- 21
- C. POWER SPECTRUM ------------------------------ 24
- D. GENERATION AND USE OF PSEUDO-RANDOM SEQUENCES FOR ADDRESSING IN RADA SYSTEMS -- 24

IV. DETECTION PROCESS ------------------------------- 27

- A. DESCRIPTION OF THE MODEL -------------------- 27
- B. SIGNAL POWER AT THE OUTPUT OF THE CORRELATOR ---------------------------------- 28
- C. NOISE POWER AT THE OUTPUT OF THE CORRELATOR ---------------------------------- 29
- D. DECISION SCHEMES ---------------------------- 31

V. COMPUTER SIMULATION OF THE DETECTION PROCESS --- 37

- A. GENERAL -- 37
- B. GENERATION OF THE SIGNAL AND NOISE ---------- 38
- C. SIMULATION RESULTS -------------------------- 40
  - 1. Noise Power at the Input of Decision Device ------------------------- 40
  - 2. Probability Distribution Amplitude of the Noise at the Input of the Decision Device ------------ 43
  - 3. Probability of error -------------------- 49

VI. TRANSMISSION OF INFORMATION BY PCM AND M-ary SYSTEM ------------------------------------ 53

A. PCM SYSTEM ---------------------------------- 53

B. M-ary SYSTEM -------------------------------- 58

C. EFFECT OF BANDWIDTH OF THE SYSTEMS ON THE NUMBER OF SIMULTANEOUS TALKERS ---------- 62

VII. BLOCK ENCODING OF INFORMATION ------------------- 65

VIII. CONVOLUTIONAL ENCODING OF INFORMATION ----------- 72

IX. CONCLUSIONS AND RECOMMENDATIONS FOR THE FUTURE ------------------------------------ 79

COMPUTER PROGRAMS ------------------------------------- 81

LIST OF REFERENCES ------------------------------------ 84

INITIAL DISTRIBUTION LIST ----------------------------- 87

FORM DD 1473 -- 89

LIST OF TABLES

Table

I Calculated and Simulated Values of the Noise Power at the Input of the Decision Device ----- 44

II Calculation of the Chi-square Function for the Noise Formed by the Transmissions of 4 Simultaneous Talkers ------------------------ 46

III Calculation of the Chi-square Function for the Noise Formed by the Transmissions of 20 Simultaneous Talkers ----------------------- 47

IV Variation of M with Frame-by-frame Encoding and Different Values of Error-correcting Ability --------------------------------------- 69

V Variation of M, for Different Values of Information Digits, and 1 Error-correction Ability --------------------------------------- 69

LIST OF FIGURES

Figure

1 General Model of RADA Systems ------------------ 17

2 Typical Pseudo-Random Sequence of 31 Digits ------------------------------------ 21

3 Autocorrelation Function of Truly Random Sequence ------------------------------------ 23

4 Autocorrelation Function of Finite Pseudo-Random Sequence ------------------------------ 23

5 Five-stage Linear Shift Register --------------- 25

6 Model of the Detection Process ----------------- 27

7 Decision Scheme for On-off Systems ------------- 34

8 Decision Scheme for Bipolar Systems ------------ 35

9 Model of the Process Simulated in a Digital Computer ------------------------------ 37

10 Shift Register Generator of the Pseudo-random Sequence Used as Signal --------------- 38

11 Shift Register Generator of the Pseudo-random Sequence Used for Generating Noise -- 39

12 Probability of Error Curve for On-off Systems Compared with the Simulated Results ------------------------------------- 51

13 Probability of Error Curve for Bipolar Systems Compared with the Simulated Results ------------------------------------- 52

14 Modulation Diagram ----------------------------- 55

15 Probability of Error for On-off and Bipolar PCM System --------------------------- 57

16 Model of M-ary System -------------------------- 59

17 Probability of Error for M-ary System Compared with the Bipolar PCM System --------- 61

18 Variation of the Number of Talkers with Bandwidth in a M-ary System ------------------ 64

19 Probability of Error for PCM Block-Encoded System with 1, 2, and 4 Error-Correcting Ability -- 67

20 Probability of Error for 1 Error-Correcting and 1, 3 and 6 Frames of Delay --------------- 70

21 Probability of Error for 2/3 Rate Convolutional Code System with 40 and 60 Digits of Constraint Length ------------------ 77

LIST OF SYMBOLS AND ABBREVIATIONS

Symbols

E_i	Expected value of the i^{th} sample
e	Number of errors corrected
$G_x(f)$	Power spectral density function of x(t)
h(t)	Impulse response
k	Number of information digits
L	Number of simultaneous transmissions. Number of talkers
M	Number of pulses composing a pseudo-random sequence
M_M	Number of pulses of the pseudo-random sequence used as address in M-ary system
M_{PCM}	Number of pulses of the pseudo-random sequence used as address in PCM system
m	Number of check digits
N	Number of stages of a shift register. Constraint length in convolutional codes
n	Total number of digits. n = m+k
n(t)	Waveform that represents noise
$\overline{n^2(t)}$	Average noise power
$n_1(t)$	Noise at the input of the correlation detector
$n_2(t)$	Noise at the output of the multiplier of the correlation detector
$n_o(t)$	Noise at the input of the decision device
O_i	Observed frequency of the i^{th} sample
P(x)	Probability of x
P(x/y)	Conditional probability of x, given y

$p(x)$	Probability density function of x
$R_x(\tau)$	Autocorrelation function of x
S	Signal power
$s(t)$	Waveform that represents the signal
$s_1(t)$	Signal at the input of the correlation detector
$s_2(t)$	Signal at the output of the multiplier of the correlation detector
$s_o(t)$	Signal at the input of the decision device
T	Time duration of an individual pulse of a pseudo-random sequence
$u(t)$	Unit step function
$v(t)$	Pseudo-random sequence locally generated in the detector
W	Bandwidth
$x_s(t)$	Pseudo-random sequence used as address of the desired transmission
$x_i(t)$	Pseudo-random sequence used as address of the i^{th} transmission
β	Channel error probability
γ^2	Signal-to-noise power ratio
$\delta(t-t_1)$	Unit impulse function
$\phi(x)$	Characteristic polynomial
μ	Normalized variable
χ^2	Chi-square function

Abbreviations

dB	Decibel
dc	Direct current
FDM	Frequency division multiple access
Fig.	Figure

H_o	Null hypothesis
H_1	Alternative hypothesis
kHz	Kilohertz
MHz	Megahertz
No.	Number
PCM	Pulse code modulation
RADA	Random access discrete address
RF	Radio frequency
TDM	Time division multiple access
t	Time
μs	Microseconds

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I. INTRODUCTION

The development of communication satellites is a fast growing area. Among other advantages, they can provide many communication channels between widely separated points. The repeater, at high altitudes is visible in large portions of the earth, and potentially can interconnect a large number of users. A new technique, called multiple access has been developed. It can be defined as the capability that all the component stations of the satellite net can communicate with each other, directly and simultaneously.

The limitations of space and weight of the satellite makes it undesirable for it to incorporate complicated switching centrals and other techniques are necessary. Using the concepts of multiplexing, Frequency Division Multiple Access (FDMA) and Time Division Multiple Access (TDMA) have been proposed [Refs. 1-9]. FDMA is the most straightforward method, and uses the existing technology and hardware. It consists in dividing the available bandwidth into frequency slots and assigning each slot to one of the stations. Undesirable intermodulation products limit this assignment and also cause inefficient use of the spectrum. TDMA overcomes most of those difficulties, but the problem of time synchronization of the whole net is not trivial. In this system the available time must be divided into slots, and each recurrent slot is assigned to one of the stations composing the net. All the stations must be able

to be synchronized between them to avoid interferences, this requirement being more strict as the number of users increases.

A different philosophy for communications had been proposed to accomplish multiple access; it has been named Random Access Discrete Address (RADA) [Ref. 1-9]. This system permits direct access between users, via a common frequency band, and eliminates the necessity of time synchronization of the net. Also individual frequency assignments to the users are not necessary. For proper identification between users there must be some initial agreement or addressing. One proposed form of addressing, used in digital communication systems, is the delay between the information pulse and a replica [Ref. 4]. Another scheme used for addressing is to divide the available bandwidth into a few frequency slots (that assures no intermodulation products) and to divide the available time into slots. To each station is assigned an address, (a matrix of frequency and time, selected in some specific form) attempting to get as many orthogonal addresses as possible [Refs. 5, 10]. Due to its orthogonality, the sequences can be transmitted without the need of synchronization of the net. With the careful selection of frequencies the disadvantages of intermodulation products have been overcome. The same idea of forming orthogonal addresses can be made possible with only one frequency and selecting sequences of pulses in the time domain which have an orthogonal property. These

sequences are called pseudo-noise signals or pseudo-random sequences. In this study, the use of pseudo-random sequences for addressing is related to various digital forms of transmission of the information.

However, one of the characteristics of the RADA system is that another noise source is introduced, namely that formed by all the simultaneous transmissions. In this thesis it is assumed that the efficiency of the system is limited by this type of interference.

In satellite systems, the use of an active repeater is essential for improving the reliability of the communications. Limitations of the size and weight for the satellite compel the optimum use of the available power. In the actual technology, one of the best microwave power amplifiers for this purpose is the traveling wave tube, having the characteristic that its optimum operating point is near saturation. But this region exhibits undesirable properties, such as non-uniform gain vs frequency, producing cross-talk interference between users. One way to avoid this problem is to use a hard limiting stage in the repeater. Studies of the effect on the reception, for systems under this condition, have been reported [Refs. 1-3]. In this thesis the study was performed for a linear case, without hard limiting. In this form the results can be applied to other systems that have the same needs of direct access, such as tactical or vehicular communication systems.

A summary of the contents of the thesis follows: Section II explains the general communication model which is analyzed in the rest of the sections. Section III gives a review of the fundamental characteristics of the pseudo-random sequences used in later analysis. Section IV refers to the problem of detection of a particular pseudo-random sequence, contaminated by other sequences. A useful formula is derived which relates signal-to-noise ratio at the output of the correlation detector with length of the sequences and the number of simultaneous transmissions. Due to the importance of these relations for the results of later part of the thesis, a digital computer simulation of the process is presented in Section V. Sections VI, VII, and VIII discuss several approaches for the transmission of information. The probability of error in the detection of the pseudo-random sequence is calculated for the following modulation methods: pulse code modulation without and with block and convolutional encoding, and M-ary systems. Interesting limitations peculiar to RADA systems are discussed. Section IX gives final conclusions and recommendations for further research in this area.

II. DESCRIPTION OF THE MODEL FOR THE SYSTEM

A general model for the RADA system is shown in Figure 1. This model suggests that each of the L + 1 active transmitters sends its information to the desired receiver via a common channel, using a common frequency. Such a RADA scheme is possible without any overall synchronization of the system, as will be explained later.

For a proper reception of the desired transmission, the transmitter and receiver must have a previous code for identificating themselves, and this code also must be such as to allow rejection of unwanted signals in the receiver. This technique is known as addressing, and has given rise to different ideas presented in References 1-12.

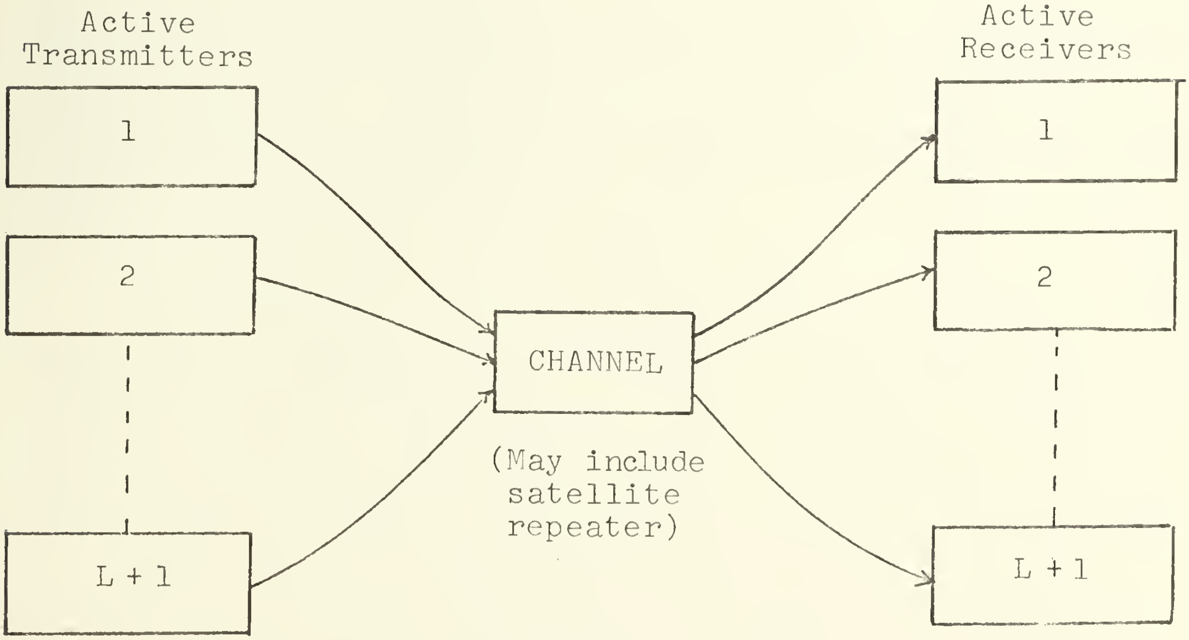

Figure 1. General Model of RADA System.

The type of addressing used in this thesis was to assign to each transmitter and receiver a different pseudo-random sequence. The transmitter, to indicate its presence, must send the address of the desired receiver, and as information, its own address. With a suitable signaling system, the receiver accepts, if it is not busy, the address of the transmitter and after that both are locked.

The presence of the address itself can represent a pulse of information, or the information may be embedded in the shift of the sequence, depending on the particular system. System details will be analyzed in later sections.

The channel, for a tactical or vehicular type of communication system, is a linear summer of the desired transmission, with the noise formed by the linear sum of all the other simultaneous talkers. Each one has the same carrier, but different phase delay, using at the radio-frequency end of the receiver a phase quadrature type of detection for eliminating the carriers, before detecting the individual sequence in the correlation detector.

For a satellite system, the active repeater can be considered as a component of the channel. Supposing two frequency bands in this scheme, one the up-link carrier, is detected by the repeater for all the transmissions reaching the satellite; after amplification, the composed signal, formed by the linear sum of all the sequences, modulates the down-link carrier using the second frequency band. This signal reaches all the receivers. Each receiver performs a

radio-frequency detection process and, by correlation methods, the pseudo-random sequence characteristic of the desired transmission is detected.

In the following Sections the process of detection of the sequence and the different possible forms of conveying information are explained.

III. PROPERTIES OF THE PSEUDO-RANDOM SEQUENCES

A. GENERAL

Pseudo-random sequences are based in the statistical behaviour of random events. A careful examination of sequences of random events, such as the results of throwing an honest coin, on the average presents some regularities: same probability of heads as tails, shorter runs of heads or tails are more probable than the longer ones, and all different sequences are uncorrelated. The autocorrelation also is zero, except for $|\tau| < T$ shift.

For electrical signals, say a sequence of bipolar pulses, these characteristics can be exploited in various ways. In this thesis pseudo-random sequences are used as addresses in the RADA system.

In order that information can be carried by each pseudo-random sequence, it may be "modulated" in the following manner: If the information is in the form of a series of binary pulses (1's and 0's or marks and space), a mark is transmitted by sending the whole pseudo-random sequence unchanged. If a space is to be sent, the pseudo-random sequence is inverted (bipolar method) or not sent (on-off method).

Figure 2 shows a typical pseudo-random sequence composed of 31 digits. This sequence is almost of zero mean; of 16 runs 8 are of length 1, 4 of length 2, 2 of length 3, 1 of length 4 and 1 of length 5. A test of autocorrelation shows

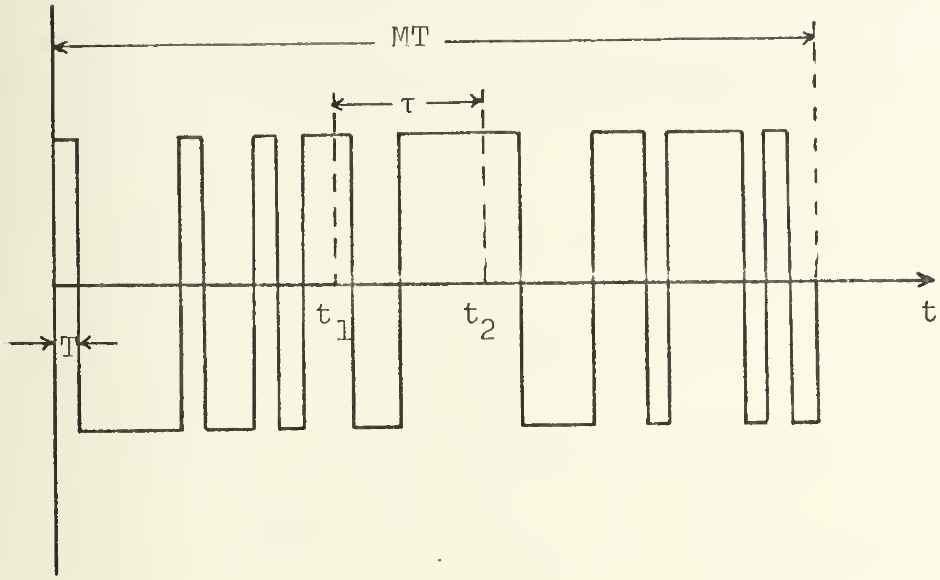

Figure 2. Typical Pseudo-random
Sequence of 31 Digits.

a constant value of -1/31 at any time shift τ different from $|\tau| < T$. For $\tau = 0$, the autocorrelation is maximum, of course.

The notation for defining the sequences, useful for future analysis, are:

T = duration of each pulse in μs.

MT = duration of the whole sequence in μs.

B. AUTOCORRELATION FUNCTION

The autocorrelation function for a truly random sequence of pulses of the type shown in Fig. 2 is:

$$R_x(\tau) = \Sigma\, x_1 x_2\, p(x_1, x_2; \tau);\ x_1 = x(t_1),\ x_2 = x(t_2),\ \tau = t_2 - t_1 \tag{3.1}$$

$$R_x(\tau) = 1\cdot 1\cdot P(1,1)+(-1)(-1)P(-1,-1)+(-1)(1)P(-1,1)$$
$$+(1)(-1)P(1,-1)$$

$$= P(1,1)+P(-1,-1)-P(-1,1)-P(1,-1) \qquad (3.2)$$

Using the relations:

$$\begin{aligned} P(1,1) &= P(1)P(1/1) \\ P(-1,1) &= P(-1)P(1/-1) = P(1)P(-1/1) \\ P(1,-1) &= P(1)P(-1/1) \\ P(-1,-1) &= P(-1)P(-1/-1) = P(1)P(1/1) \end{aligned} \qquad (3.3)$$

and noting that P(1) = P(-1) = 1/2 the autocorrelation becomes:

$$R_x(\tau) = P(1/1) - P(-1/1)\ . \qquad (3.4)$$

These conditional probabilities are:

$$P(1/1) = \begin{cases} 1 - |\tau|/2T & ; \quad |\tau| \leq T \\ 1/2 & ; \quad |\tau| > T \end{cases} \qquad (3.5)$$

$$P(-1/1) = \begin{cases} |\tau|/2T & ; \quad |\tau| < T \\ 1/2 & ; \quad |\tau| > T \end{cases} \qquad (3.6)$$

The autocorrelation is then

$$\begin{aligned} R_x(\tau) &= 1 - |\tau|/T & ; \quad |\tau| \leq T \\ &= 0 & ; \quad |\tau| \geq T \end{aligned} \qquad (3.7)$$

and is shown in Fig. 3.

For certain types of periodic sequences of length M, of the type shown in Fig. 2, the autocorrelation for $|\tau| > T$ is not zero, but is a small amount, of value -1/M and is shown in Fig. 4. This value is almost negligible for large

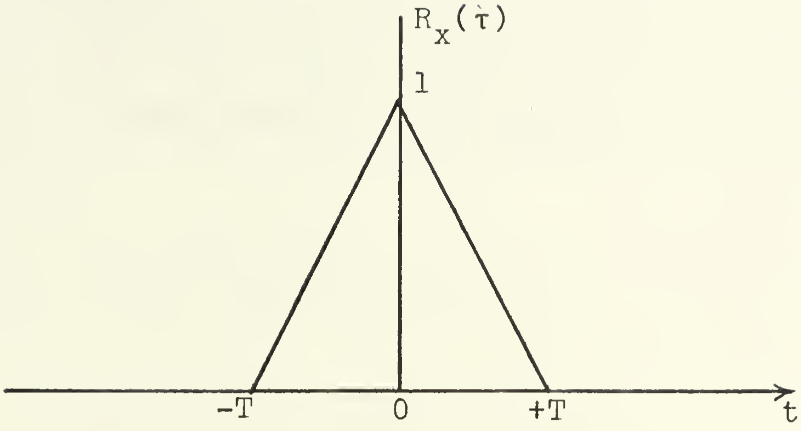

Figure 3. Autocorrelation Function of Truly Random Sequence.

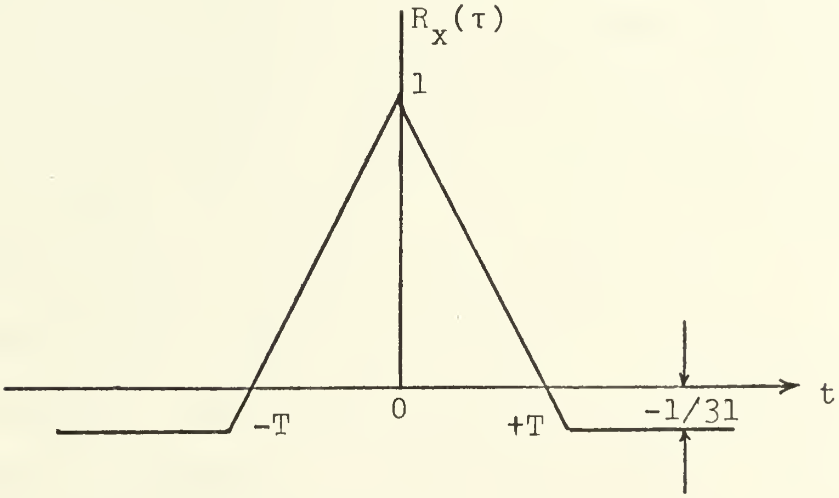

Figure 4. Autocorrelation Function of Finite Pseudo-random Sequences.

values of M. The example uses the sequence of Fig. 2, where the autocorrelation is -1/31 for $|\tau| > T$. [Ref. 13].

C. POWER SPECTRUM

The power spectrum, $G_x(f)$ is the Fourier transform of the autocorrelation function, and considering this as an even function,

$$G_x(f) = 2 \int_0^\infty R_x(\tau) \cos \omega\tau \, d\tau \tag{3.8}$$

$$= 2 \int_0^\infty (1 - \tau/T) \cos \omega\tau \, d\tau$$

$$= T\left(\frac{\sin \omega T/2}{\omega T/2}\right)^2 \quad . \tag{3.9}$$

D. GENERATION AND USE OF PSEUDO-RANDOM SEQUENCES FOR ADDRESSING IN RADA SYSTEMS

Not all sequences generated in some random experiment have the characteristics defined. More than that, is not easy to select at random a specific sequence with all those properties. One may recall that those properties, commonly named as balance, run and correlation are characteristics of the whole ensemble of sequences, but not all the components of the ensemble must necessarily meet them.

Different methods have been devised for generating specific sequences that assure having certain desirable requirements of randomness. One of the simplest methods uses the sequences generated by a shift register, to which has been added modulo-two adders, whose output is used as

feedback to the first stage of the register. Fig. 5 shows the connections of a 5-stage shift register and the pseudo-random sequence taken from the last stage, when the initial value of the register is 00001, as shown below.

For the example shown the length of the sequence is equal to $2^N - 1$, N being the number of stages of the register. In this example $2^N - 1$ is 31.

The connections of the shift register can be precisely determined from relations developed in linear algebra. Tables have been developed that aid in this determination [Refs. 13-14].

Under certain conditions the register will have maximal period so that the same sequence will be generated regardless

1000010010110011111000110111010

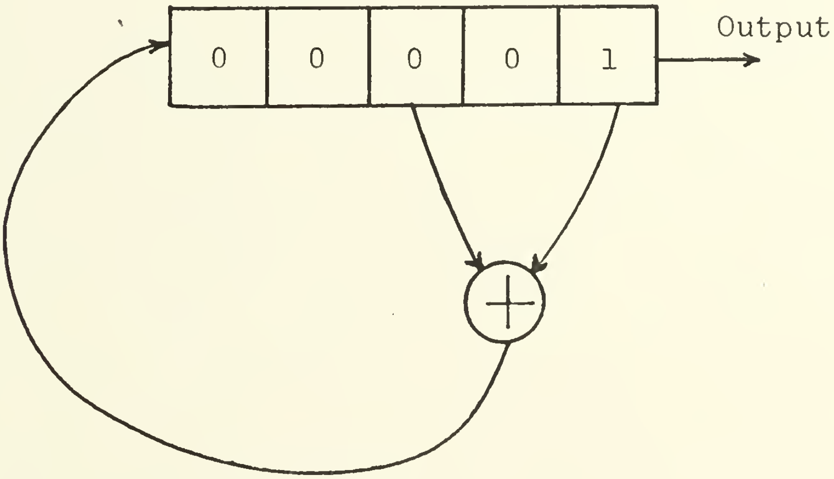

Figure 5. Five-stage Linear Shift Register.

of the initial contents of the stage of the register [Ref. 13].

Unfortunately a simple application of this idea is not useful for addressing in the RADA systems, because of the limited number of different pseudo-random sequences, of moderate length, produced for a given N- stage shift register.

One successful approach consists in using a shift register that produces a very long sequence before repeating, (e.g. a 19-stage shift register), which can be divided into non-overlapping segments, using each segment as an address. Each segment does not necessarily meet the properties of randomness, and a careful search must be made to obtain desirable sub-sequences. Corr et al [Ref. 15] explain an approach, using computer programming for selecting suitable sequences. One example in their paper was the selection of 1000 uncorrelated segments, of length 63, from a 17-stage register.

IV. DETECTION PROCESS

A. DESCRIPTION OF THE MODEL

At the input of the receiver of Figure 6 the signal may be expressed as:

$$s(t) = 2x_s(t) \cos \omega_o t \tag{4.1}$$

where $x_s(t)$ is the pseudo-random sequence that phase-reversal modulates the carrier. The noise is represented by:

$$n(t) = 2 \sum_{i=1}^{L} x_i(t) \cos \omega_o t \ . \tag{4.2}$$

This noise is formed by the linear sum of L other transmitters operating simultaneously. These formulas suppose a satellite link, as explained in Section II, and $\cos \omega_o t$ represents the common down-link carrier. For other systems, where all the carriers have the same frequency but arrive at

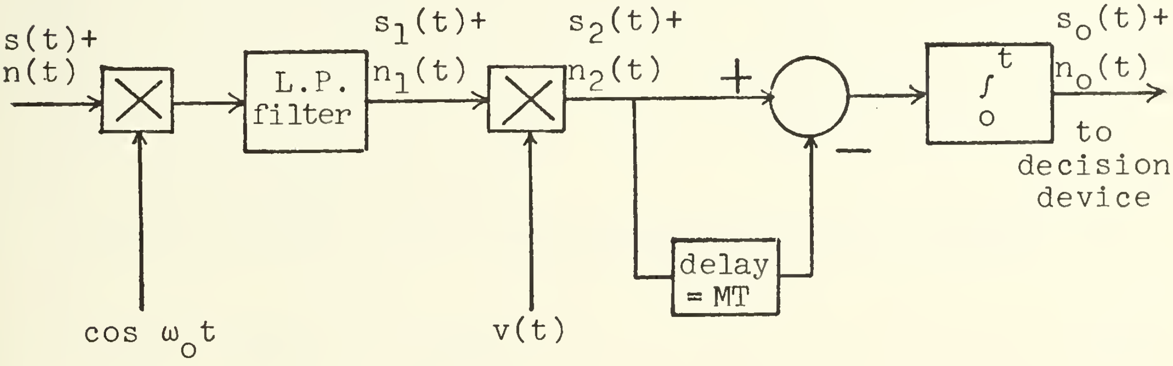

Figure 6. Model of the Detection Process.

the receiver with different phase, a successful approach is to use a phase quadrature detector. In this scheme, the local oscillator of the receiver generates cos $\omega_o t$ and sin $\omega_o t$. The components, coming from respective mixers, are added in order to obtain the detected signals embedded in the carriers. This arrangement takes care of the unknown phase of the signal, obtaining the same final results explained for the satellite case. However, if it is possible to predict in advance the phase of the desired signal, a 3-dB increase in the signal-to-noise ratio will be obtained.

B. SIGNAL POWER AT THE OUTPUT OF THE CORRELATOR

The signal s(t), after detection and low-pass filtering becomes:

$$s_1(t) = x_s(t) . \tag{4.3}$$

This signal is multiplied by the same locally generated address sequence v(t), obtaining a dc value of ±1. Note that $x_s(t)$ is equal to v(t) multiplied by +1 or -1, depending on whether a mark or a space was transmitted by bipolar modulation, or by +1 or 0 for on-off modulation of a mark or a space, respectively.

At the output of the averaging device the signal is:

$$s_o(t) = \frac{1}{MT} \int_o^{MT} dt = 1 \tag{4.4}$$

where MT is the time duration of the sequence.

The signal power is S = 1.

C. NOISE POWER AT THE OUTPUT OF THE CORRELATOR

After detection and low-pass filtering the noise is:

$$n_1(t) = \sum_{i=1}^{L} x_i(t) \; . \tag{4.5}$$

Multiplying by v(t) one obtains:

$$n_2(t) = v(t) \sum_{i=1}^{L} x_i(t) \; . \tag{4.6}$$

This is the noise present at the input of the averaging device. This averaging device has the impulse response:

$$h(t) = \frac{1}{MT} [u(t) - u(t-MT)] \; , \tag{4.7}$$

where u(t) is the unit step function.

The average power at the output is

$$\overline{n_o^2(t)} = \iint h(\rho)\, h(\sigma)\, R_{n_2}(\sigma-\rho)\, d\sigma\, d\rho \; . \tag{4.8}$$

$R_{n_2}(\sigma-\rho)$ is the autocorrelation function of the noise, $n_2(t)$, at the input to the averaging filter. Let $\sigma-\rho = \lambda$.

In order to find $\overline{n_o^2(t)}$, one must obtain $R_{n_1}(\lambda)$, the autocorrelation function of $n_1(t)$, in order to obtain $R_{n_2}(\lambda)$.

Calculating $R_{n_1}(\lambda)$, the autocorrelation of $n_1(t)$:

$$R_{n_1}(\lambda) = \overline{n_1(t)n_1(t+\lambda)} = \overline{\sum_{i=1}^{L} x_i(t) \sum_{i=1}^{L} x_i(t+\lambda)}$$

$$= [\overline{x_1(t)x_1(t+\lambda)}+\ldots+\overline{x_L(t)x_L(t+\lambda)}+\overline{\text{CROSS TERMS}}]$$

$$= L\, R_x(\lambda) \; . \tag{4.9}$$

These results use the orthogonality property of the sequences. The autocorrelation of $n_2(t)$ is obtained from:

$$R_{n_2}(\lambda) = \overline{n_2(t)n_2(t+\lambda)} = \overline{v(t)v(t+\lambda)\Sigma x_i(t)\Sigma x_i(t+\lambda)} \quad (4.10)$$

Since all the sequences are statistically independent,

$$\begin{aligned} R_{n_2}(\lambda) &= \overline{v(t)v(t+\lambda)}\ \overline{\Sigma x_i(t)\Sigma x_i(t+\lambda)} \\ &= R_x(\lambda)\ R_{n_1}(\lambda) \\ &= L\ R_x^2(\lambda)\ . \end{aligned} \quad (4.11)$$

Using the results of Section III, where $R_x(\lambda)$ denotes the autocorrelation of a single pseudo-random sequence:

$$R_x(\lambda) = (1 - |\lambda|/T) \quad 0 \leq |\lambda| \leq T\ . \quad (4.12)$$

The autocorrelation of $n_2(t)$ is:

$$R_{n_2}(\lambda) = L(1 - |\lambda|/T)^2 \quad (4.13)$$

The average power at the output becomes:

$$\begin{aligned} \overline{n_o^2(t)} &= \int_o^{MT}\int_o^{MT} \frac{L}{(MT)^2}\left(1 - \frac{|\lambda|}{T}\right)^2 d\rho\, d\sigma \\ &= \frac{L}{(MT)^2}\int_o^{MT}\int_{-\rho}^{MT-\rho} \left(1 - \frac{|\lambda|}{T}\right)^2 d\lambda\, d\rho;\ \rho \geq 0 \end{aligned} \quad (4.14)$$

Call

$$g(\rho) = \int_{-\rho}^{MT-\rho} \left(1 - \frac{|\lambda|}{T}\right)^2 d\lambda\ . \quad (4.15)$$

The evaluation of this integral is:

$$g(\rho) = \begin{cases} \frac{T}{3}\,(2 - (1 - \frac{\rho}{T})^3) & ; \quad 0 \le \rho \le T \\ \frac{2}{3}T & ; \quad T \le \rho \le (M-1)T \\ \frac{T}{3}\,(2 - (\frac{\rho}{T} - (M-1))^3 & ; \quad (M-1)T \le \rho \le MT \end{cases} \qquad (4.16)$$

With these results the noise power is:

$$\overline{n_o^2(t)} = \frac{L}{(MT)^2}\left[2 \int_o^T \frac{T}{3}\,(2 - (1-\frac{\rho}{T})^3)\,d\rho + \int_T^{(M-1)T} \frac{2}{3}T\,d\rho\right]$$

$$= \frac{2L}{3M}\,(1 - \frac{1}{4M}) \;. \qquad (4.17)$$

For values of M normally used ($M \gg 1$), a good approximation of the noise power, at the output of the correlation detector, is:

$$\overline{n_o^2(t)} \simeq \frac{2}{3}\,\frac{L}{M} \;. \qquad (4.18)$$

D. DECISION SCHEMES

At the input of the receiver, the noise is formed by the linear addition of the addresses of the unwanted transmissions. With the considerations that the sequences are statistically independent and are formed by rectangular pulses, the probability distribution of the amplitude of the noise is binomial, with zero mean. Given L independent signals, each one equal to +1 or -1 at a given instant with equal probability,

let y = sum of the L signals. Then

y = (No. of +1's) - (No. of -1's)

$y = u - (L - u)$, where u = No. of +1's

or $y = 2u - L$, so

$$u = \frac{L+y}{2} \; . \tag{4.19}$$

Since the probability that y assumes a certain value is the probability that it assumes the value given by the last equation, then

$$P[n(t) = y] = \binom{L}{\frac{L+y}{2}} \left(\frac{1}{2}\right)^L \tag{4.20}$$

This binomial distribution is of course for ideal rectangular signals, but if the signals are bandlimited, or have slight variations of amplitude, this distribution is smoothed, becoming continuous and approaching the normal distribution. This effect is more pronounced for large values of L, being an application of the central limit theorem.

The next step is the analysis of the effect of the correlation process in the statistics of the noise. The first component of the correlator is the multiplier. At its output the noise, denoted now by z(t), is:

$$z(t) = n(t)v(t) \; . \tag{4.21}$$

Considering v(t) and n(t) to be statistically independent random variables, the probability density of their product can be found from: [Ref. 17].

$$p(z) = \int_{-\infty}^{+\infty} \frac{1}{|v|} \frac{e^{-\frac{z^2}{2\sigma^2 v^2}}}{2\sqrt{2\pi}\,\sigma} [\delta(v-1) + \delta(v+1)] \, dv, \tag{4.22}$$

it being considered in this expression that

$$p[v(t) = v] = \frac{1}{2} [\delta(v-1) + \delta(v-1)] \qquad (4.23)$$

is the probability distribution of the pseudo-random sequence $v(t)$, and that the noise $n(t)$ is gaussian distributed, of zero mean and variance σ^2.

Performing the indicated integration one obtains:

$$p(z) = \frac{1}{\sqrt{2\pi}\ \sigma} e^{-\frac{z^2}{2\sigma^2}} \qquad (4.24)$$

which tells that the probability distribution of the noise at the output of the multiplier is also gaussian. The following stage of the correlator is an averaging filter. This is a linear element, and in consequence, the noise signal out is also gaussian.

Therefore at the input to the decision device, the noise is gaussian distributed, with zero mean and variance σ_o^2:

$$p(n) = \frac{1}{\sqrt{2\pi}\ \sigma_o^2} e^{-\frac{n^2}{2\sigma_o^2}} \quad ; \quad \sigma_o{}^2 = \frac{2}{3}\frac{L}{M} \ . \qquad (4.25)$$

The decision device compares two hypotheses, for deciding if the signal received at the sampling time corresponds or not to the information sent by the transmitter. Further, two different methods of signal modulation are considered; one, an on-off type, and the other a bipolar type.

The first decision scheme is shown in Fig. 7 and the hypotheses to be tested are:

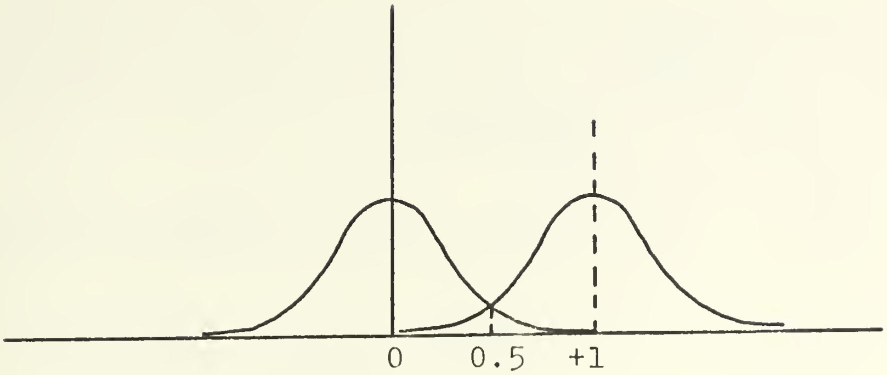

Figure 7. Decision scheme for on-off systems.

$H_o = x = n$

$H_1 = x = s + n$

If x = n,

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma_o} e^{-\frac{x^2}{2\sigma_o^2}} \qquad (4.26)$$

σ_o^2 being the average power of the noise.

If x = s + n,

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma_o} e^{-\frac{(x-1)^2}{2\sigma_o^2}} \qquad (4.27)$$

For a symmetric channel, the threshold for x must be selected at the intersection of the two curves, corresponding to x = 1/2.

A wrong detection is performed when the noise is over the threshold:

$$P(\text{error}) = p(n > \tfrac{1}{2}) = \int_{\frac{1}{2}}^{\infty} \frac{1}{\sqrt{2\pi}\sqrt{\frac{2L}{3M}}} e^{-\frac{3x^2M}{4L}} dx \qquad (4.28)$$

Letting γ^2 be signal to noise ratio,

$$\gamma \triangleq \sqrt{\frac{S}{\sigma_o^2}} = \sqrt{\frac{1}{\frac{2L}{3M}}} = \sqrt{\frac{3}{2}\frac{M}{L}} \ . \qquad (4.29)$$

Let $y = \frac{x}{\sqrt{\frac{2}{3}\frac{L}{M}}}$;

Then $P(\text{error}) = \int_{.5\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \ . \qquad (4.30)$

For the bipolar type of detection, represented in Figure 8 the hypotheses to be tested are:

$$H_o = x = s + n$$
$$H_1 = x = -s + n$$

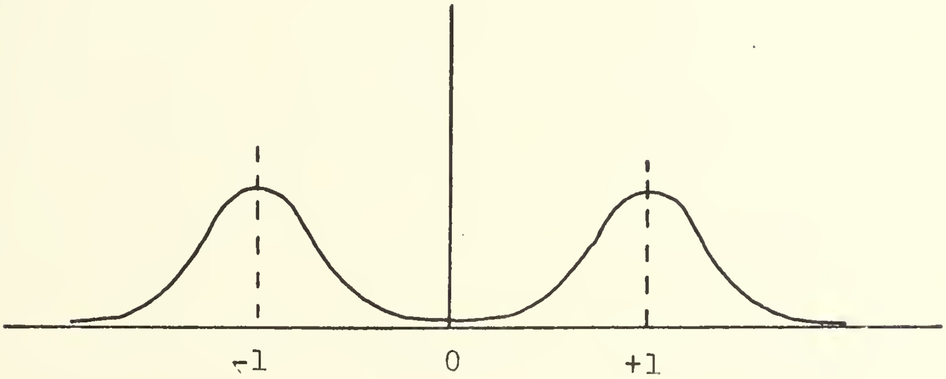

Figure 8. Decision Scheme for Bipolar Systems.

Their probability distributions are: if $x = s + n$,

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma_o} e^{-\frac{(x-1)^2}{2\sigma_o^2}} , \qquad (4.31)$$

and if $x = -s + n$

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma_o} e^{-\frac{(x+1)^2}{2\sigma_o^2}} . \qquad (4.32)$$

In this case the threshold is 0, and following the same approach as before, the probability of error becomes:

$$P(\text{error}) = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy . \qquad (4.33)$$

These two formulas are basic for the treatment of the following sections.

V. COMPUTER SIMULATION OF THE DETECTION PROCESS

A. GENERAL

Due to the importance of the theoretical results for the later stages of the thesis, the detection process was simulated on an IBM-360 digital computer. The model studied in the simulation is shown in Fig. 9. The receiver used for the local signal v(t) a pseudo-random sequence of 127 digits for purposes of correlation. This same sequence is supposed to be sent by the transmitter and corrupted by additive noise, representing the effect of the channel. The correlation receiver makes a decision every 127 samples if the received signal was noise or signal plus noise, using a threshold type of decision process. The threshold

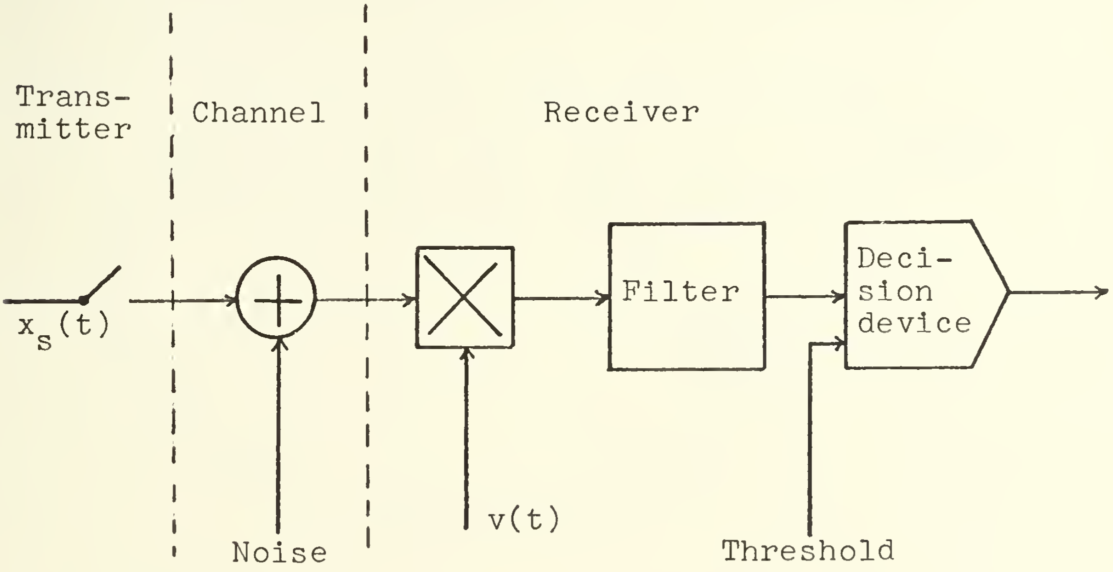

Figure 9. Model of the Process Simulated in a Digital Computer.

level was selected for simulating bipolar and on-off types of detection. For getting acceptable statistics for the process and still keeping the computer time under reasonable limits, 500 iterations were performed for different number of simultaneous talkers. The statistics obtained were noise power, mean and amplitude probability distribution of the noise at the input of the decision device, and channel error probability.

B. GENERATION OF THE SIGNAL AND NOISE

The signal used was the pseudo-random sequence generated by the 7-stage linear shift register shown in Fig. 10, corresponding to the characteristic polynomial $\phi(x) = 1+x^2+x^7$. The output values were registered on data cards; each digit when stored in the computer was given the value +1 or -1, as specified.

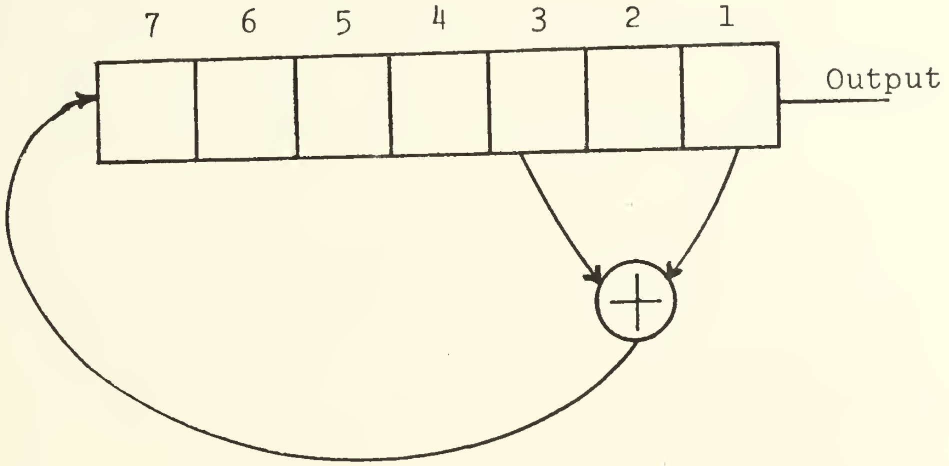

Figure 10. Shift Register Generator of the Pseudo-random sequence used as signal.

Two different approaches were used to simulate the noise. The first consisted in generating gaussian noise, using for this effect a subroutine obtained from the scientific package of the library of the IBM system. This subroutine generates normally distributed floating point numbers, of specified mean and variance, by the method of summing 12 random uniformly distributed numbers [Ref. 18].

The second approach used was to generate the noise by the summing of random sequences, similar to the actual case studied. The sequence used for generating the noise is shown in Fig. 11 and corresponds to the polynomial $\phi(x) = 1 + x + x^7$.

This sequence was made different from the one used as the signal in order to avoid the possibility that in any iteration the sequence used as signal could be analyzed as noise.

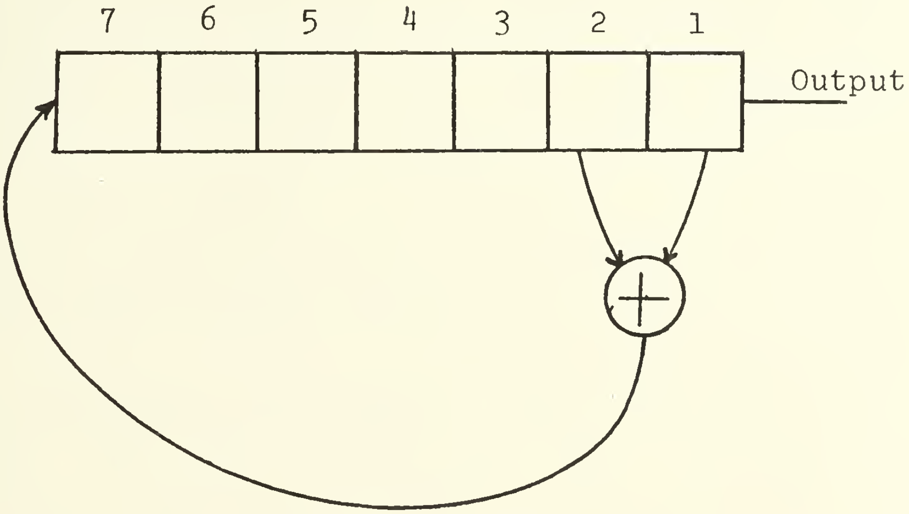

Figure 11. Shift Register Generator of the Pseudo-random Sequence Used for Generating Noise.

In the simulation of the effect of several talkers, shifted sequences represented addresses of the different talkers, taking advantage of the negligible crosscorrelation of the sequences. The amount of shift of each sequence during each iteration was a uniformly distributed discrete random variable ranging between 1 and 126 pulse times. Also, for more realistic effect, the amplitude coefficient of each sequence was permitted to have slight changes; this amplitude variation for each one of the talkers had a normal distribution of mean 1.0 and standard deviation 0.07.

C. SIMULATION RESULTS

1. Noise Power at the Input of the Decision Device

In the treatment of the analytical problem in Section IV the sequences were considered with zero autocorrelation, for $|\tau| > T$. This assumption is only an approximation, and permits analytical simplifications. But in real systems or simulations the finite autocorrelation terms should affect the results. It was stated (see Section III) that the autocorrelation of a sequence is finite and of value $-1/M$, M being the number of pulses in the sequence. This same result is also true for the crosscorrelation of different pseudo-random sequences for any value of τ [Ref. 13]. With this consideration, the expression for the noise power at the input of the decision device will be derived again.

From Section IV, the value of the autocorrelation of the noise at the input of the correlator was given as:

$$R_{n_1}(\lambda) = [L\ R_x(\lambda) + \overline{\text{CROSS TERMS}}] \qquad (5.1)$$

where L is the number of interfering talkers: $R_x(\lambda)$ is the autocorrelation of the input signal.

In the former derivation the cross terms were considered as zero. Considering now only this term, denoted as $R_{n_{cl}}(\lambda)$,

$$R_{n_{cl}}(\lambda) = [\overline{\text{CROSS TERMS}}] = \overline{\sum_{i=1}^{L} \sum_{j=1}^{L} x_i(t) x_j(t+\lambda)}$$

$$= [L(L-1)\overline{x_i(t) x_j(t+\lambda)}]. \qquad (5.2)$$

Recalling that

$$\overline{x_i(t) x_j(t+\lambda)} = \frac{1}{M}$$

$$R_{n_{cl}}(\lambda) = \frac{L(L-1)}{M}\ . \qquad (5.3)$$

Also the autocorrelation of the noise $n_2(t)$, at the input of the averaging filter after being multiplied by v(t) was given by:

$$R_{n_2}(\lambda) = \overline{v(t)v(t+\lambda)}\ \overline{\sum_{i=1}^{L} x_i(t) \sum_{i=1}^{L} x_i(t+\lambda)}\ , \qquad (5.4)$$

using the fact that the sequences are statistically independent. In the present derivation $R_{n_2}(\lambda)$ becomes:

$$R_{n_2}(\lambda) = L\ R_x^{\ 2}(\lambda) + R_x(\lambda)\ R_{ncl}(\lambda). \qquad (5.5)$$

The second term is due to the finite crosscorrelation. Denote this term as $R_{n_{c2}}(\lambda)$;

$$R_{n_{c2}}(\lambda) = R_x(\lambda)\, R_{n_{c1}}(\lambda)$$

$$= (1 - \frac{|\lambda|}{T})\ \frac{L(L-1)}{M}\ . \qquad (5.6)$$

The noise power term at the output of the averaging filter $\overline{n_c^2(t)}$, due to the finite crosscorrelation is:

$$\overline{n_c^2(t)} = \int_0^{MT} \int_{-\rho}^{MT-\rho} \frac{L(L-1)}{M}\ \frac{1}{(MT)^2}\ (1 - \frac{|\lambda|}{T})\ d\lambda\ d\rho \qquad (5.7)$$

with $\sigma - \rho = \lambda$, $d\sigma = d\lambda$.

$$\text{Call} \quad g_1(\rho) = \int_{-\rho}^{MT-\rho} (1 - \frac{|\lambda|}{T})\ d\lambda \qquad (5.8)$$

and with the consideration that the interesting range of ρ is $0 \leq \rho \leq MT$, the value of $g_1(\rho)$ is:

$$g_1(\rho) = \begin{cases} T - \frac{T}{2}(1 - \frac{\rho}{T})^2 & ;\ 0 \leq \rho \leq T \\ T & ;\ T \leq \rho \leq (M-1)T \\ T(1 - \frac{1}{2}(\frac{\rho}{T} - (M-1))^2 & ;\ (M-1)T \leq \rho \leq MT \end{cases} \qquad (5.9)$$

Again, with good approximation, for $M \gg 1$, $g_1(\rho)$ can be considered a constant of value T, in the interval $0 \leq \rho \leq MT$.

With these results, the noise power due to the cross term is

$$\overline{n_c^2(t)} = \frac{L(L-1)}{M^3T^2} \int_0^{MT} T \, d\rho$$

$$= \frac{L(L-1)}{M^2} , \qquad (5.10)$$

and the total noise power at the input of the decision device is

$$\overline{n_o^2(t)} = \frac{2L}{3M} + \frac{L(L-1)}{M^2} . \qquad (5.11)$$

With this formula, the noise power was calculated for different numbers of talkers and for both cases, namely finite as well as zero crosscorrelation. These values are presented in Table I and are compared with results obtained from the simulation.

The results of the simulation agree very closely with the theoretical results for finite crosscorrelation. Also, the results of both methods of simulating noise justify the assumption that the noise can be considered to have a gaussian distribution. This is shown with more rigorous arguments later in this section.

For shorter sequences, and a large number of talkers, the results for the noise power differ appreciably depending on whether the cross terms are considered or neglected. This fact must be taken into account in the possible application of the results.

2. Probability Distribution of the Amplitude of the Noise at the Input to the Decision Device

In Section IV the amplitude distribution of the noise at the input to the decision device was discussed and

TABLE I

CALCULATED AND SIMULATED VALUES OF THE NOISE POWER AT THE INPUT OF THE DECISION DEVICE

Number of talkers, L	Calc. Noise Power		Noise Power of Simul.	
	with zero crosscorr.	with finite crosscorr.	with gauss. noise	with sum of seq.
2	0.0105	0.0106		0.0129
3	0.0157	0.0161		0.0213
4	0.0210	0.0217	0.0327	0.0388
6	0.0315	0.0336	0.0491	0.0472
9	0.0472	0.0517	0.0736	0.0694
12	0.0630	0.0704	0.0982	0.0966
20	0.1045	0.1280	0.1637	0.1708
25	0.1312	0.1684	0.2046	0.1956
35	0.1837	0.2575	0.2865	0.2731
50	0.2625	0.4144	0.4093	0.408

it was stated that the noise could be considered gaussian, of zero mean, and having standard deviation equal to the square root of the noise power. With this result, the decision strategy was selected that would maximize the probability of detection and therefore minimize the error rate. In the simulation it was desired to check that last statement, so a histogram was analyzed for two cases: few talkers and many simultaneous talkers. This histogram was performed in the part of the simulation where the noise is

formed by the addition of pseudo-random sequences, for the sum of 4 and the sum of 20 of them.

The test of consistency between the hypothetical gaussian distribution and distribution observed from the samples was performed using the chi-square goodness-of-fit test. The null hypothesis, that the sample distribution agrees with the gaussian distribution, was tested.

For 4 talkers, the sample mean and the standard deviation σ, obtained from the simulation results are:

Mean = 0.10

σ = 0.19

With these values the normalized variable, μ, becomes:

$$\mu = \frac{n(t) - \overline{n(t)}}{\sigma} = \frac{n(t) - 0.1}{0.19} \qquad (5.12)$$

where n(t) is the amplitude of the noise, and $\overline{n(t)}$ its average value.

The χ^2 function is defined as:

$$\chi^2 = \sum_i \frac{(0_i - E_i)^2}{E_i} \qquad (5.13)$$

0_i is the observed frequency of the i^{th} sample and E_i is the expected frequency.

In Table II are shown the steps necessary for the calculation of χ^2 along with the observed and estimated results of the histogram.

The degree of freedom used is determined by the number of samples (in this case 21) reduced by 1. Since the mean

TABLE II

CALCULATION OF THE CHI-SQUARE FUNCTION FOR THE NOISE FORMED BY THE TRANSMISSIONS OF 4 SIMULTANEOUS TALKERS

Limit values	μ for limits	Area 0 to μ	Area for interval	Exper. freq.	Observed freq.	$\frac{(O_i-E_i)^2}{E_i}$
-0.525	3.29	0.4995				
			0.0008	0	0	0
-0.475	3.02	0.4987				
			0.0016	1	1	0
-0.425	2.76	0.4971				
			0.0033	2	0	2
-0.375	2.5	0.4938				
			0.0063	3	1	1.33
-0.325	2.24	0.4875				
			0.0116	6	3	1.50
-0.275	1.975	0.4759				
			0.0195	10	3	4.9
-0.225	1.71	0.4564				
			0.0310	15	10	1.66
-0.175	1.445	0.4254				
			0.0434	22	16	1.63
-0.125	1.185	0.3820				
			0.0608	30	23	1.63
-0.075	0.92	0.3212				
			0.0759	38	39	0.02
-0.025	0.659	0.2453				
			0.0918	46	50	0.34
0.025	0.395	0.1535				
			0.1006	50	55	0.5
0.075	0.133	0.0529				
			0.1058	53	62	1.52
0.125	0.133	0.0529				
			0.1006	50	58	1.28
0.175	0.395	0.1535				
			0.0918	46	55	1.76
0.225	0.659	0.2453				
			0.0759	38	45	1.28
0.275	0.92	0.3212				
			0.0608	30	35	0.83
0.325	1.185	0.3820				
			0.0434	22	22	0
0.375	1.445	0.4254				
			0.0310	15	12	0.6
0.425	1.71	0.4564				
			0.0195	10	6	1.6
0.475	1.975	0.4759				
			0.0116	6	3	1.5
0.525	2.24	0.4875				

$\chi^2 = 25.88$

TABLE III

CALCULATION OF THE CHI-SQUARE FUNCTION FOR THE NOISE FORMED BY THE TRANSMISSIONS OF 20 SIMULTANEOUS TALKERS

Limit values	μ for limits	Area 0 to μ	Area for interval	Exper. freq.	Observed freq.	$\frac{(O_i-E_i)^2}{E_i}$
-1.05	2.8	0.4974				
			0.0023	1	1	0
-0.95	2.58	0.4951				
			0.0053	3	3	0
-0.85	2.32	0.4898				
			0.0090	5	4	0.20
-0.75	2.07	0.4808				
			0.0144	7	4	1.28
-0.65	1.83	0.4664				
			0.0235	12	9	0.75
-0.55	1.58	0.4429				
			0.0330	17	17	0
-0.45	1.34	0.4099				
			0.0456	23	22	0.04
-0.35	1.10	0.3643				
			0.0581	29	30	0.03
-0.25	0.854	0.3062				
			0.0771	39	35	0.41
-0.15	0.61	0.2291				
			0.0855	43	43	0
-0.05	0.368	0.1436				
			0.0938	47	50	0.19
0.05	0.124	0.0498				
			0.0996	50	44	0.72
0.15	0.124	0.0498				
			0.0938	47	53	0.76
0.25	0.368	0.1436				
			0.0855	43	45	0.09
0.35	0.61	0.2291				
			0.0771	39	42	0.23
0.45	0.854	0.3062				
			0.0581	29	30	0.03
0.55	1.10	0.3643				
			0.0456	23	25	0.17
0.65	1.34	0.4099				
			0.0330	17	17	0
0.75	1.58	0.4429				
			0.0235	12	13	0.08
0.85	1.83	0.4664				
			0.0144	7	7	0
0.95	2.07	0.4808				
			0.0090	5	3	0.80
1.05	2.32	0.4898				

$\chi^2 = 5.78$

and standard deviation were estimated from the sample values, there is another reduction of 2, becoming 21-1-2 = 18 degrees of freedom. For 18 degrees of freedom, and significance level of 0.05, the value of χ^2, taken from tables [Ref. 19] is 28.9, and the calculated value is 25.88. Since the calculated χ^2 from the sample values is less than that obtained from tables, there is no evidence against the hypothesis that n(t) has a gaussian amplitude distribution. For the case of 20 talkers, the standard deviation observed from the sample values was 0.41, and the mean 0.1. The normalized random variable μ was taken as $\mu = \frac{n(t) - 0.1}{0.41}$ and the level of significance of the histogram 0.1. The tabulated χ^2 obtained is 26.0 and that calculated from the sample values 5.78. Again, there is no evidence against the hypothesis that n(t) is gaussian.

The results of the observed samples were obtained from a mean of several histograms performed with different sets of sequences. This statistical test is not definite, and does not state that the samples are gaussian distributed, but at least does not reject this hypothesis. One important characteristic of the systems was not simulated, in order to save computer time, namely that the sequences are band limited. With this further smoothing effect the distribution of the noise should more closely approach being gaussian. It may be concluded that the decision scheme presented in Section IV is justified.

The sequences summed to simulate the noise had a mean value of 0.1. This value was taken into account in the results of the histogram, but no further consideration was given in the analytical treatment, where the mean was taken as zero. In an actual system the threshold level should be adjusted to suit the mean value in order to have a symmetric channel.

3. Probability of Error

The estimated probability of error, in detecting the pseudo-random sequence, was calculated using the formulas developed in Section IV.

$$P(\text{error}) = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \qquad (5.14)$$

where $\gamma = \frac{3}{2}\frac{M}{L}$, $M = 127$, for the bipolar detection scheme. When the on-off type of detection was used 0.5γ was taken as the lower limit of the integral.

Also the probability of error was calculated from the results obtained in the present Section, assuming a finite crosscorrelation, with:

$$\gamma = \sqrt{\frac{1}{\frac{2}{3}\frac{L}{M} + \frac{L(L-1)}{M^2}}} = M\sqrt{\frac{3}{3L^2 + L(2M-3)}} \qquad (5.15)$$

Both results are compared with the error rate obtained from simulation and are presented in Figures 12 and 13 for the on-off and bipolar cases respectively.

The results of the simulation agree very closely with the theoretical results derived in Section IV, when

complemented with the consideration of finite crosscorrelation calculated in this Section. Also both methods of simulation are similar in their results, especially for a large number of talkers.

Having thus found an acceptable degree of closeness between the theoretical and the simulated systems, the process of detection of the address (just described) will be related to various modes of encoding information in following sections.

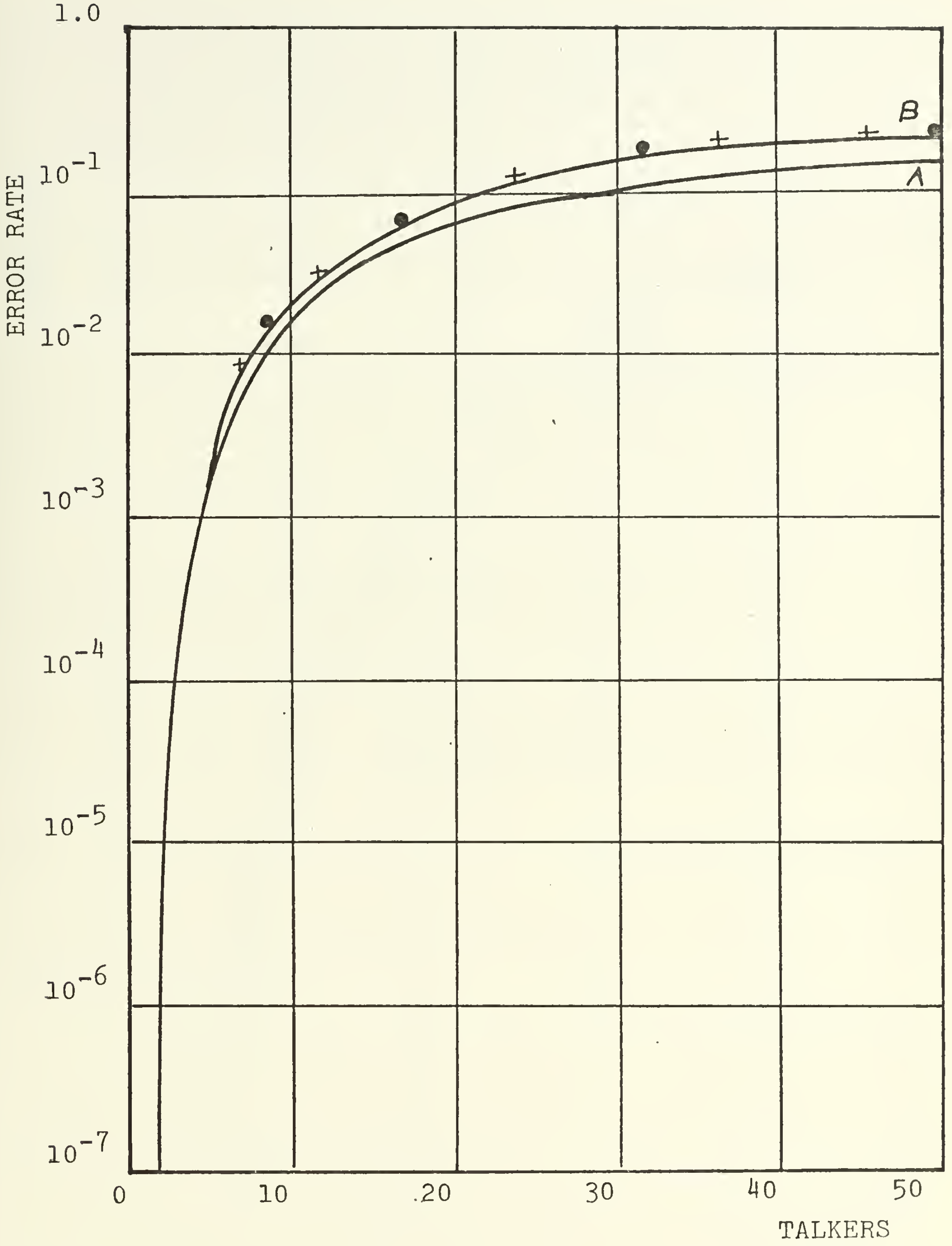

Figure 12. Probability of error curve for on-off systems compared with the simulated results. Curve A: with zero crosscorrelation. Curve B: with finite crosscorrelation. Crosses: results of simulation using sum of sequences as random noise. Dots: results using gaussian noise.

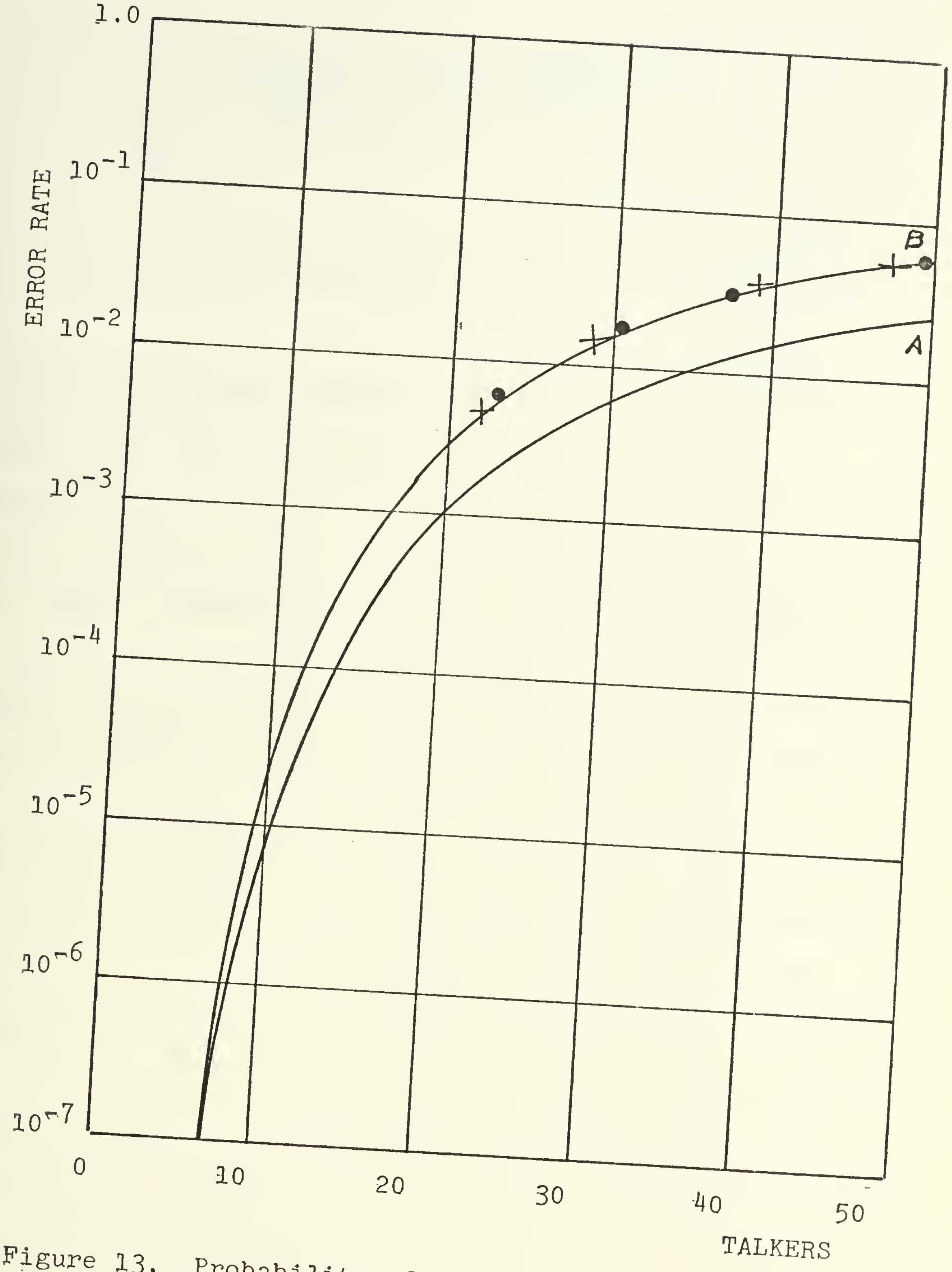

Figure 13. Probability of error curve for bipolar system compared with the simulated results. Curve A: with zero crosscorrelation. Curve B: with finite cross-correlation. Crosses: results of simulation using sum of sequences as random noise. Dots: results using gaussian noise.

VI. TRANSMISSION OF INFORMATION BY PCM AND M-ARY SYSTEMS

A. PCM SYSTEM

In earlier sections the determination of the channel error probability was developed. These results will now be related to different forms of transmission of information.

It is well known that the digital type of communications may have low signal-to-noise ratio requirement for a reasonable degree of reliability. In the heavy noisy environment, characteristic of RADA systems, this leads us to find digital methods for conveying information.

One of the obvious methods is pulse code modulation (PCM). The literature in this subject is extensive, and the principles will not be explained in this work. Bell Telephone reported a complete PCM system for high-quality voice communications [Ref. 20]. One of their conclusions, in relation to the number of quantization levels required, was that a high-quality commercial system needs no more than 128 levels for an imperceptible quantization noise. From this conclusion, the information per sample is represented by seven binary digits. Accordingly, this representation was adopted for this study. A band-limited voice signal of 4 kHz was supposed, and therefore the Nyquist sampling interval must be 125 μs. In actual systems, during that time not only the 7 information digits must be sent,

but in some cases digits may be necessary for synchronization and signaling . This depends on the particular system; in this study it is supposed that all the 125 μs are availables for information purposes.

In this study the examples consider a 10 MHz bandwidth channel. The only justification for using this figure is that it would be reasonable for a real system and it allows an easy measure for normalization, 10 MHz being a convenient reference number. The effect of variations of bandwidth in the performance of the system is discussed later in this section. In Fig. 14 is shown the modulation diagram for this system. As is explained in the figure, each pseudo-random sequence phase-reversal modulates the carrier. Phase-reversal modulation is a double-sideband, without carrier, modulation scheme, and in order to have a 10 MHz channel, each sideband must have at most 5 MHz of bandwidth. This value fixes the duration of each individual pulse of the pseudo-random sequence to 0.1 μs. In Fig. 14 it also can be seen that the allowed time slot for each pseudo-random sequence is 17.8 μs, and in consequence, the maximum number of pulses per sequence is 178. This corresponds to the M used in the description of the detection process in Section IV, and can thus be substituted in the signal-to-noise power ratio expression derived in the same section and complemented in Section V. Repeating for convenience those results, the probability of error for the PCM system is calculated by:

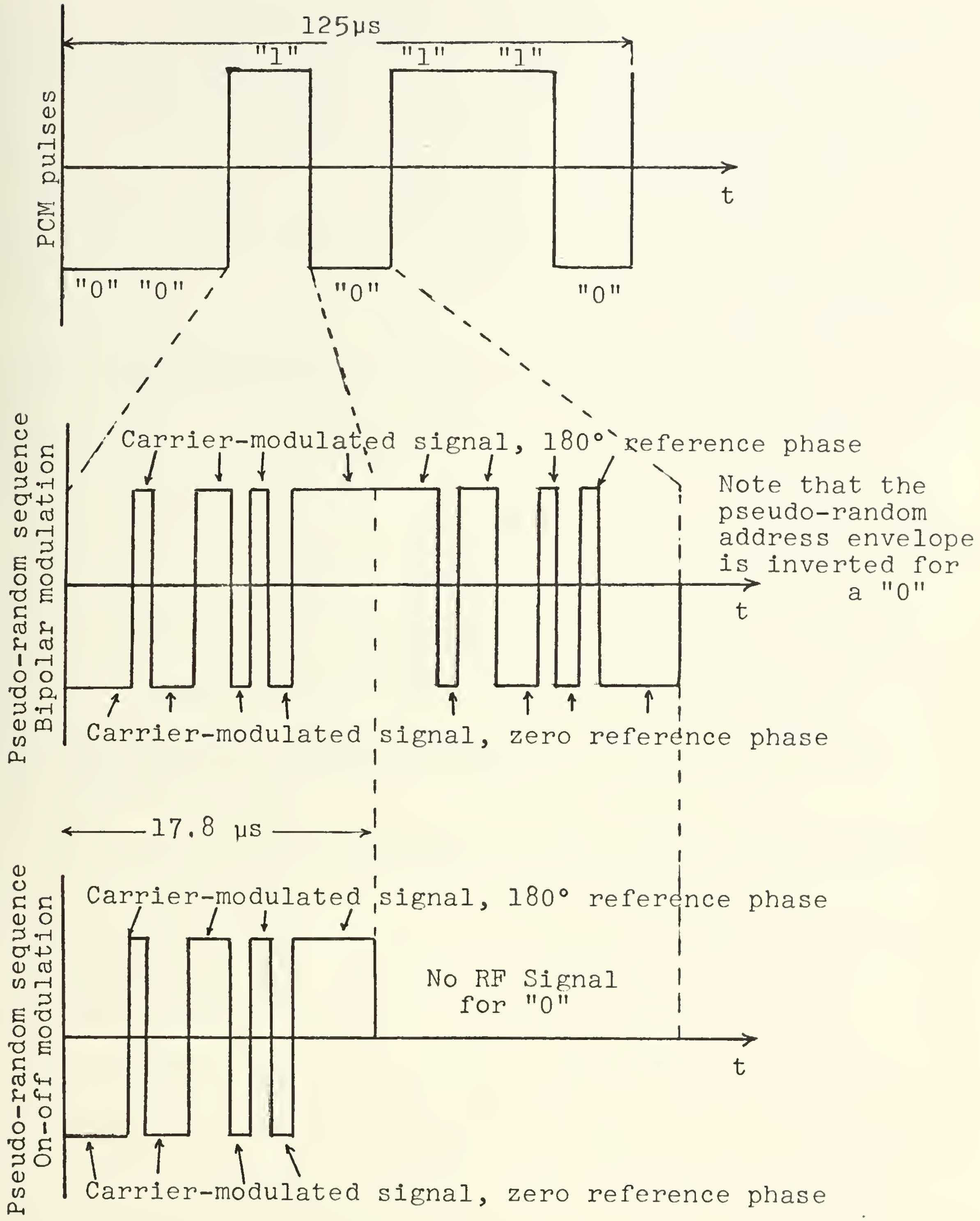
125μs
"1"
"1"
"1"
PCM pulses
t
"0"
"0"
"0"
"0"
Pseudo-random sequence
Bipolar modulation
Carrier-modulated signal, 180° reference phase
Note that the pseudo-random address envelope is inverted for a "0"
t
Carrier-modulated signal, zero reference phase
17.8 μs
Pseudo-random sequence
On-off modulation
Carrier-modulated signal, 180° reference phase
No RF Signal for "0"
t
Carrier-modulated signal, zero reference phase

Figure 14. Modulation Diagram.

$$P(\text{error}) = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \quad \text{for bipolar system, or,}$$

$$P(\text{error}) = \int_{0.5\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \text{ for on-off system.}$$

$$\gamma = M \sqrt{\frac{3}{3L^2 + L(2M-3)}} \quad \text{and} \tag{6.1}$$

$$M = \frac{1250}{7} \text{, for 10 MHz system.}$$

Both probabilities of error are evaluated, for different numbers of simultaneous transmissions, and the computer results are presented in Fig. 15. As expected, the bipolar system is superior, but what is remarkable is the great difference between the two systems in number of possible simultaneous talkers.

Despite the poor performance in the detection process of the on-off system, it offers a further advantage when it is applied to RADA systems. The on-off system, due to its nature, has a lower RF duty cycle factor than the bipolar system. This last system, has a 100% duty cycle, and the on-off, on the average, only 50%. This means that the on-off system will have half the average input noise power, compared to the bipolar case, for the same number of simultaneous talkers. Carrying this 1/2 factor thru the derivation of the error rate, for the zero crosscorrelation case, the lower limit of the integral becomes $0.5\sqrt{2}\,\gamma = .707\,\gamma$. This represents an improvement over the case shown

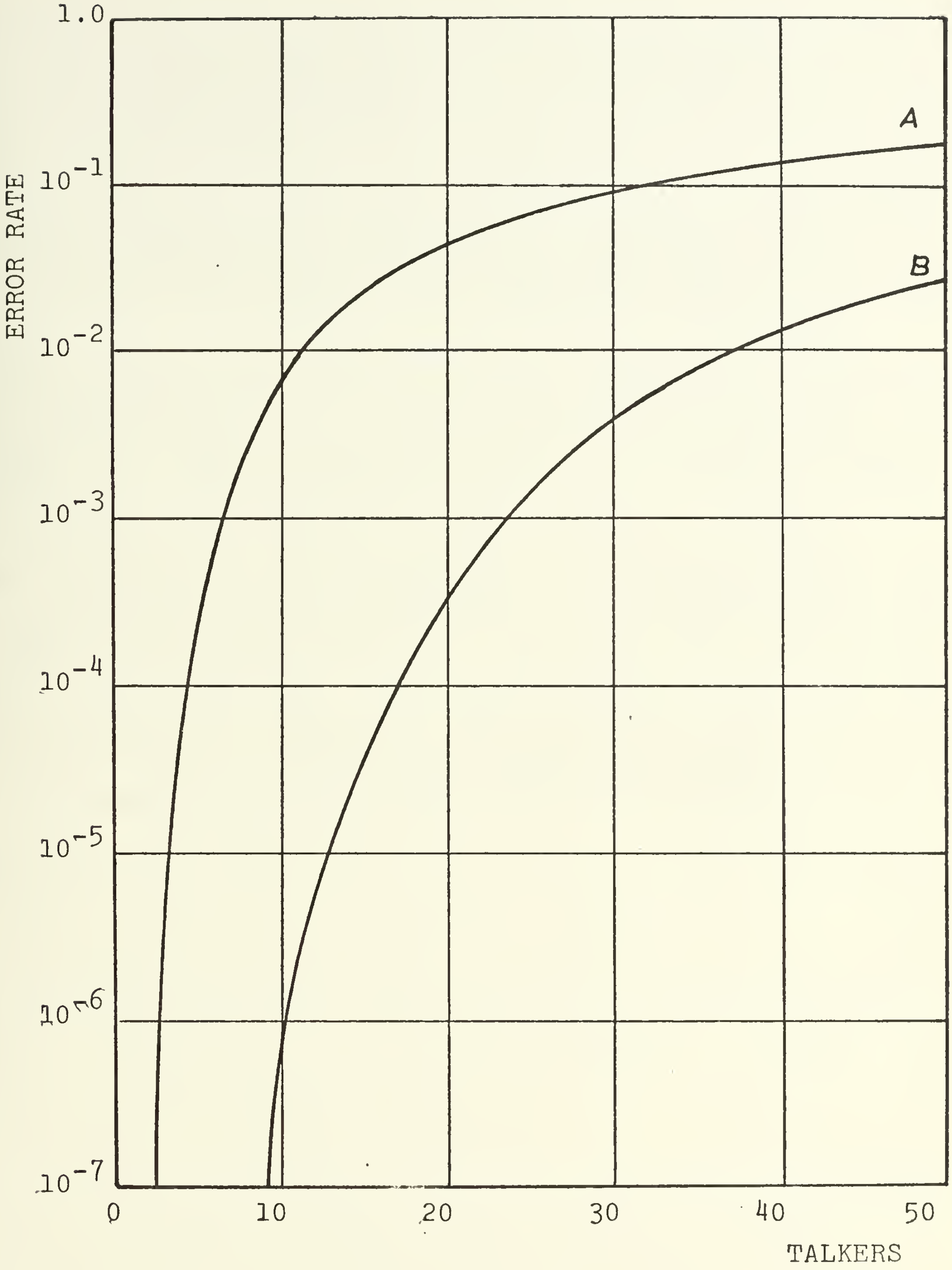

Figure 15. Probability of error for on-off and bipolar PCM system. Curve A: with on-off type of detection. Curve B: with bipolar type of detection.

in Fig. 15, for which the lower limit is only taken to be 0.5 γ, but this is still inferior to the bipolar system where the lower limit is γ.

One aspect that needs further explanation is the effect of the error rate on the reception of the voice. As is known, not all the digits of the PCM system are equally weighted, and in this case the value of each one of the seven digits is 64, 32, 16, 8, 4, 2, and 1 respectively. The loss of a digit of weight 64 produces a distinct click in the reception of the voice, but the loss of one of lower weight may not be appreciated. In 3 minutes about 10^7 digits are transmitted so if the probability of error is 10^{-7}, on the average, then one information digit will be in error every 3 minutes. It has been concluded that an exceptionally good system can permit a probability of error about 10^{-6} [Ref. 20].

B. M-ARY SYSTEM

The M-ary system has been mentioned in the literature because it makes good use of channel capacity. A model of this system is shown in Fig. 16.

Information is transmitted by the system by cyclically shifting the address (pseudo-random sequence) in time, according to the level of the quantized sample. Cyclical shifting avoids the need for a guard time interval, such as was used in Reference 15. The receiver needs 128 correlators, to decide the delay corresponding to the level of the

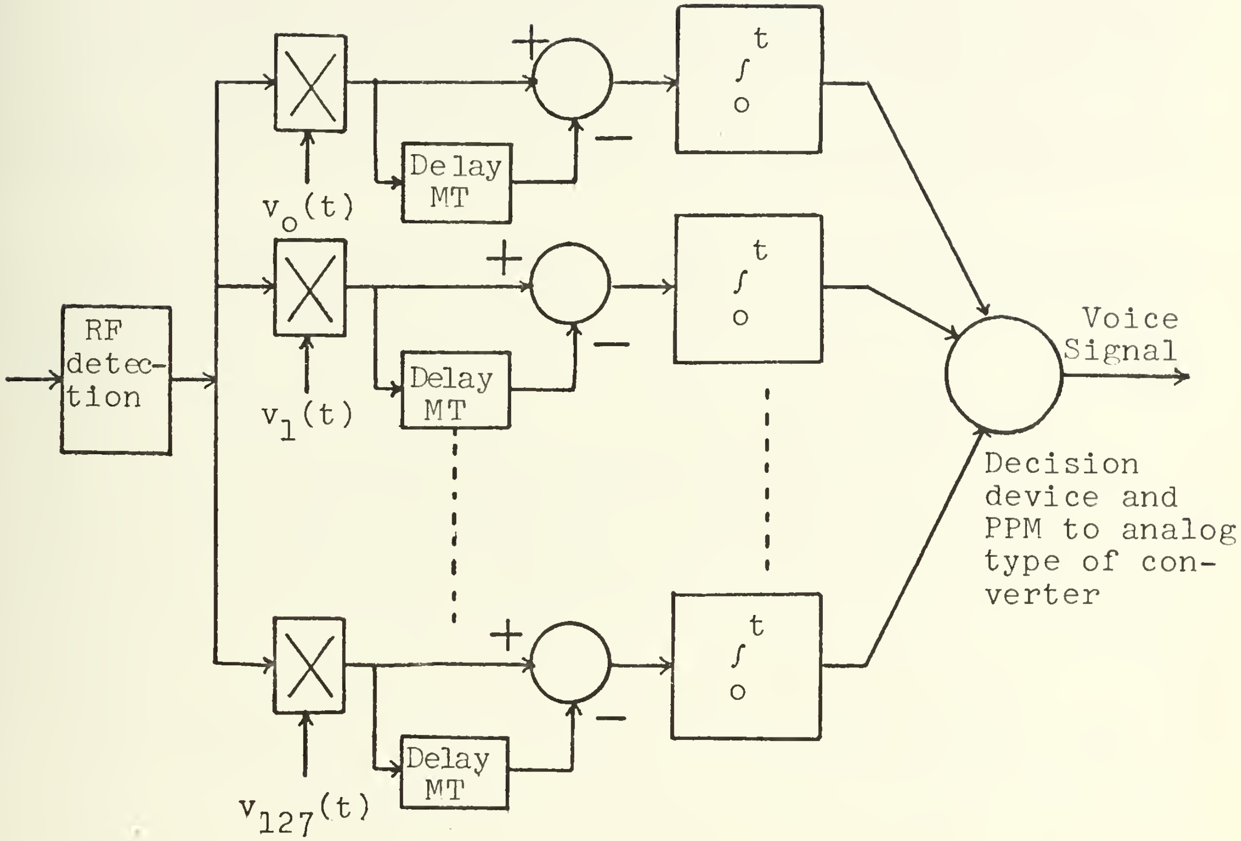

Figure 16. Model of M-ary system. $v_j(t)$ represents the basic address sequence, $v_o(t)$ cyclically shifted j times; j ranges then from 1 to 127 corresponding to the quantized level of the respective sample.

sample. This scheme unfortunately is not able to use a bipolar type of detection and also have a 100% RF duty cycle. The great advantage over the PCM system is that all the 125 μs can now be used for integration purposes in the correlation detector instead of only 17.8 μs per PCM pulse. With this consideration the value of M is 1250 (= 125/0.1) and the probability of error of the system can be calculated using:

$$P(\text{error}) = \int_{0.5\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy, \text{ where}$$

$$\gamma = M\sqrt{\frac{3}{3L^2 + L(2M-3)}}, \text{ and} \qquad (6.2)$$

$$M = 1250.$$

This formula for the probability of error is evaluated for different numbers of simultaneous transmissions and the results are shown in Fig. 17. In the same figure, for comparison, is presented the probability of error for the bipolar PCM system, showing that the M-ary system would allow more simultaneous transmissions for a given probability of error.

The explanation of the M-ary system shown in Fig. 16 is only for illustrative purposes, in order to show how its performance can be readily calculated. The difficulty in actual implementation would be that a large number of correlators is required and also the necessity for exact bit synchronization between pairs of transmissions. The same idea may be implemented using a filter matched to the basic sequence $x_s(t)$. After being fed into the filter, the received sequence is cyclically shifted until a complete match occurs and the recorded number of shifts will give the required indication of the level sent. Since the received signal sequences are continuously arriving at the filter, storage registers may be necessary. This model also needs

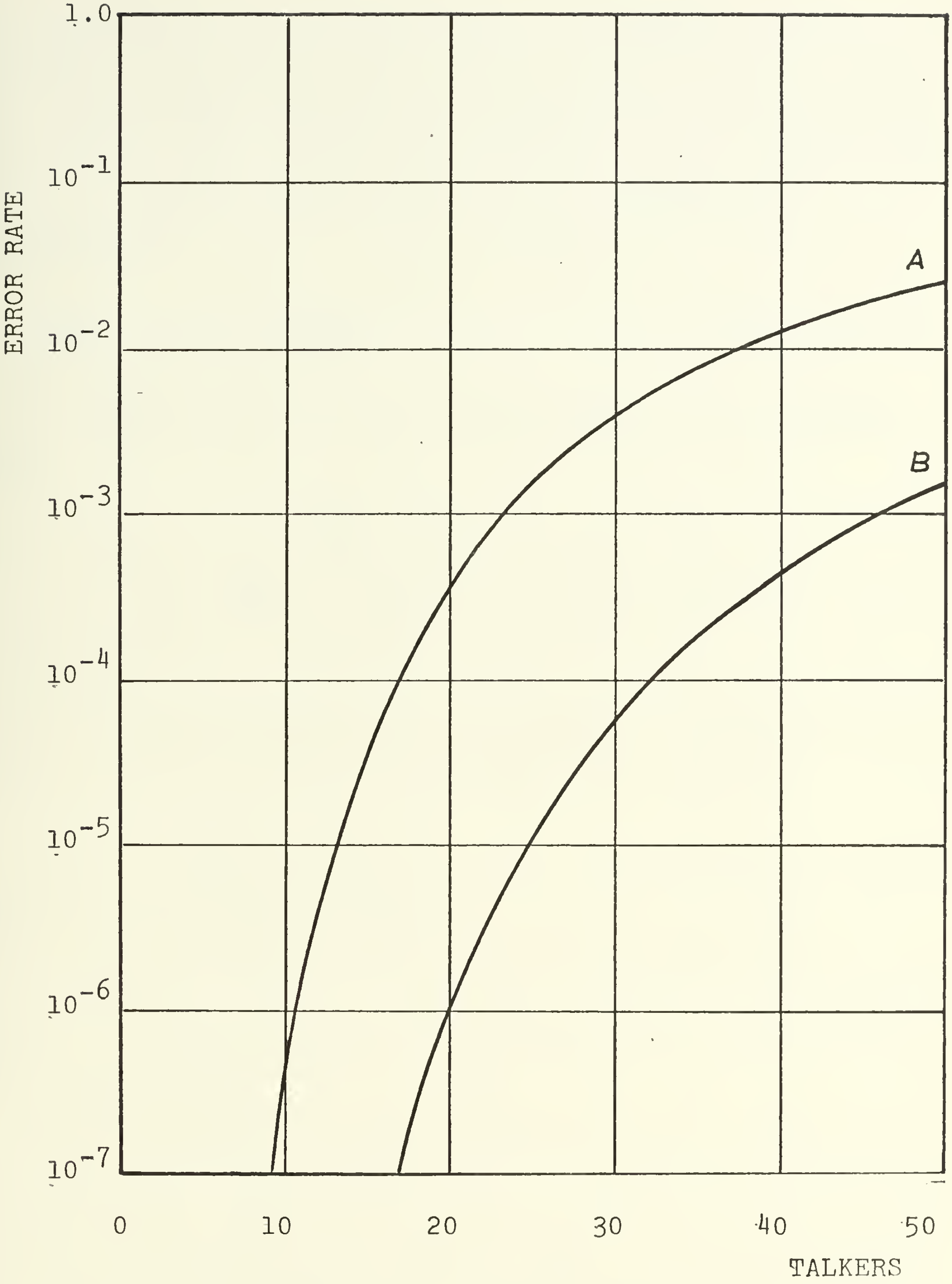

Figure 17. Probability of error for M-ary system compared with the bipolar PCM system. Curve A: PCM bipolar system, Curve B: M-ary system.

good synchronization between pairs of talkers, but it alleviates the hardware requirements of the previous model. The matched filter used in this scheme may be a tapped delay line or a digital matched filter, as explained in Reference 16.

C. EFFECT OF THE BANDWIDTH OF THE SYSTEMS IN THE NUMBER OF SIMULTANEOUS TALKERS

The general formula for β, the probability of channel error,

$$\beta = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy, \quad \text{for bipolar detection} \qquad (6.3)$$

was developed in Section III. If the detection is on-off type, 100% duty cycle, the lower limit of the integral is 0.5γ.

Since $\gamma = M\sqrt{\dfrac{3}{3L^2 + L(2M-3)}}$, β is thus a function of the number of simultaneous talkers, L, and M, the number of pulses of the pseudo-random sequence forming the receiver's address. In the PCM case, M was taken to be 178; in the M-ary case M was 1250 (≈ 7 x 178).

M can be related to the bandwidth of the RF signal as follows: each sample of the voice signal must be transmitted within an interval 125 μs long. For the PCM system this interval is divided by 7 (corresponding to 7 binary pulses per sample), and the time for transmission of one address signal, or its negative, corresponds to the time

for a binary pulse. Thus if $T = 0.1$ μs and $MT = 178 \times 0.1 = 17.8$ μs, $7 \times 17.8 = 125$ μs.

For the M-ary system, a longer address signal, cyclically shifted according to the modulating information, is transmitted for each sample. Here, if $T = 0.1$ μs, $MT = 1250 \times 0.1 = 125$ μs.

The bandwidth W is taken as:

$W = (\frac{1}{2T}) \times 2$, since both sidebands must be transmitted.

Since $M_M = 125$ μs, (M_M denoting the number of digits of the pseudo-random sequence used in M-ary system) the bandwidth of the system can be expressed as $W = \frac{M_M}{125}$. Similarly, for the PCM system, $M_{PCM} \times T = 17.8$ μs, and $W = \frac{M_{PCM}}{17.8}$ MHz.

With these expressions substituted in the channel error probability formulas, the error rate of the system can be calculated as a function of the bandwidth. The results for the M-ary system are presented in Fig. 18 showing the variation of the number of talkers with variation of bandwidth, for three different probabilities of error: 10^{-6}, 10^{-4}, and 10^{-1}. It is seen that for each value of β, the bandwidth required is proportional to the number of talkers.

This result is of course not surprising. The difficulty in the application of this result is that an increase in bandwidth will require a decrease in the duration of the individual pulses, since the sampling rate is constant. The 10 MHz bandwidth used in the examples is readily compatible with current technology. Future developments may allow a much larger bandwidth and a corresponding increase in the number of talkers.

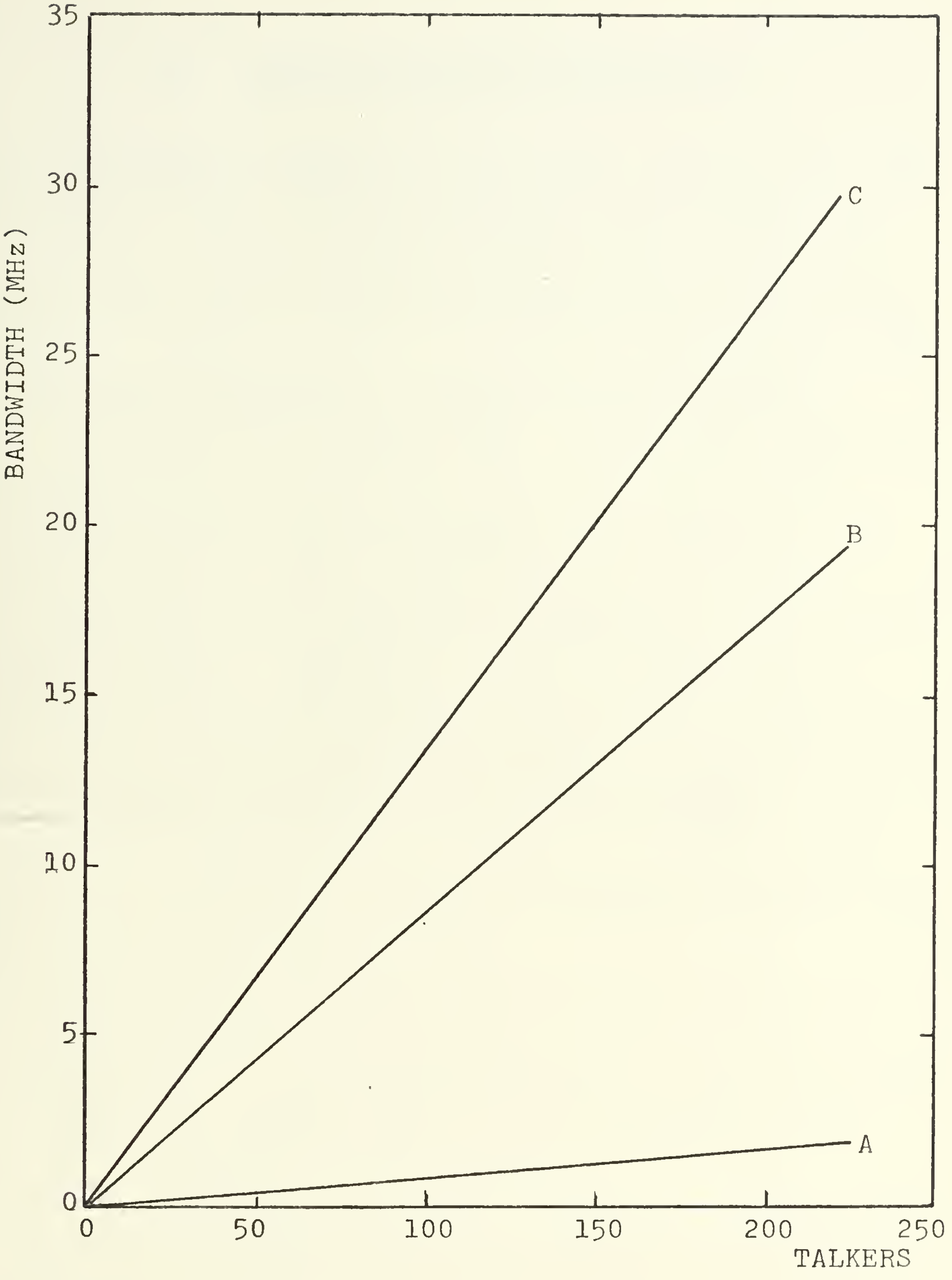

Figure 18. Variation of the number of talkers with bandwidth in a M-ary system. Curve A: $\beta = 0.1$. Curve B: $\beta = 10^{-4}$. Curve C: $\beta = 10^{-6}$.

VII. BLOCK ENCODING OF INFORMATION

The fundamental theorem for the noisy channel says that there are codes that allow transmission of information at any rate below channel capacity with an arbitrarily small probability of error. Some of the most studied codes, which also have good mathematical treatment based on modern algebra, are the cyclic block codes. It has been shown in the literature that these codes may have an arbitrarily small probability of error, determined by the number of check digits that are needed to be added to the blocks of information digits. (However, the rate of transmission of information will then be appreciably below capacity.) Using this idea, it has been proposed that coding could be one method for improving the performance of RADA systems. A feasibility study was performed to explore the effect of coding the PCM information digits.

The addressing and detection process is the same used and described in the PCM systems, but now an error-correcting encoding and decoding stage is incorporated in the transmitter and receiver, respectively.

The number of check digits required for a specified block of message digits in order to correct e-tuple errors in each coded "word" is given by the Hamming lower bound.

$$m = n - k \geq \log_2 \left(\sum_{i=1}^{e} \binom{n}{i} \right) \qquad (7.1)$$

m = number of the check digits

n = total number of digits of the coded word

k = number of information digits (7 in this application)

e = number of the digits corrected/word.

Note that this bound is a necessary but not a sufficient condition. It was adopted in the analysis in order to show the best that might be expected from an error-correcting encoding system.

The overall probability of error is given by:

$$P(\text{error}) = 1 - \sum_{i=0}^{e} \binom{n}{i} \beta^{i} (1-\beta)^{n-i} \tag{7.2}$$

where β, the channel probability of error, was calculated using:

$$\beta = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

$$\gamma = M \sqrt{\frac{3}{3L^2 + L(2M-3)}} \tag{7.3}$$

with $M = \frac{1250}{n}$. (for 10 MHz channel)

In Fig. 19 the results for 1, 2, and 4 error corrections per word are shown. Each word contains the 7 message digits required per sample via PCM. The results are far from the expected, and the block codes do not give a noticeable improvement in the system. This apparently contradictory effect is explained on the basis that the channel error

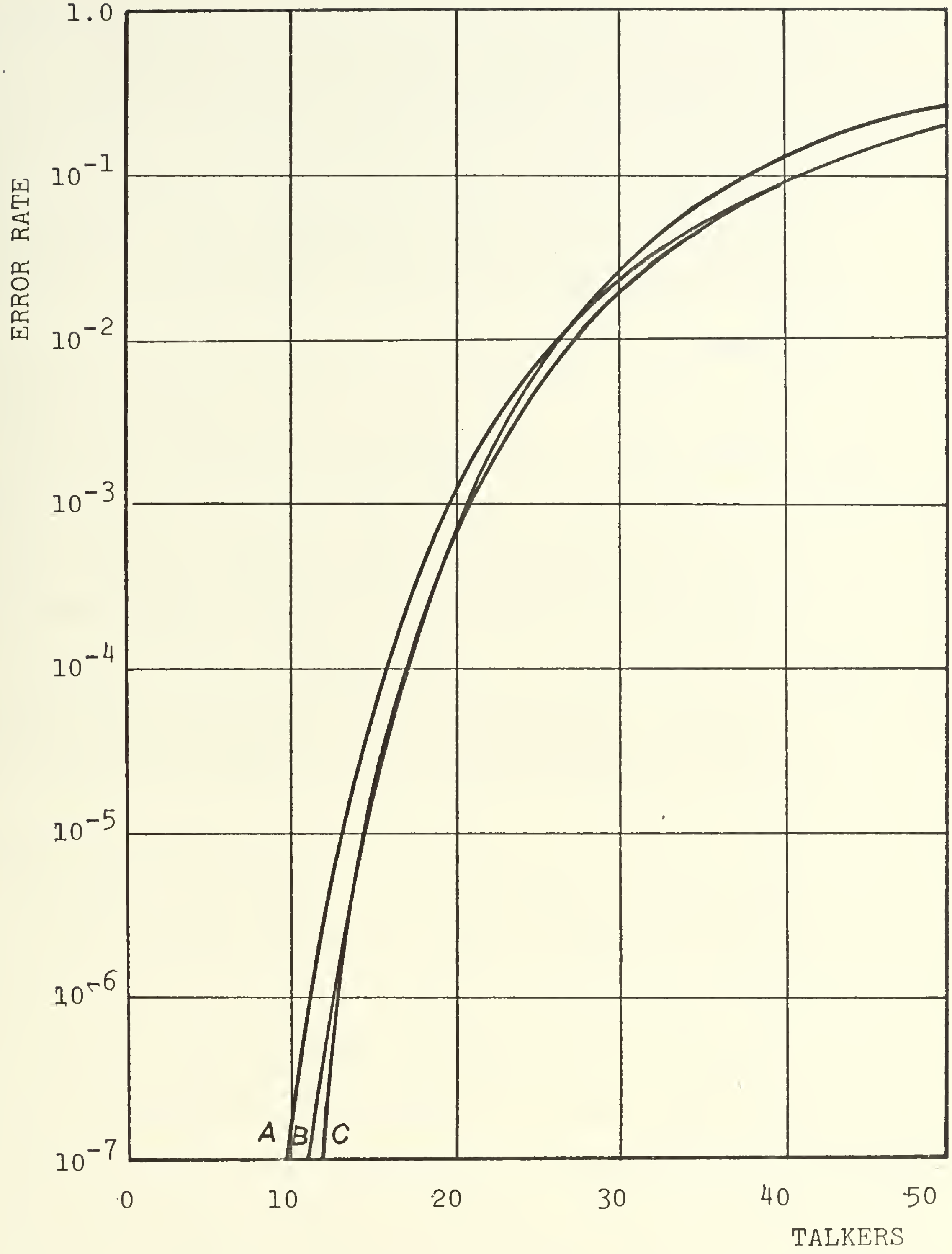

Figure 19. Probability of error for PCM block encoded system with 1, 2 and 4 error-correcting ability. Curve A: 1 error correction. Curve B: 2 error-correction. Curve C: 4 error-correction.

probability is not constant, being a function of the value of M. As more error-correcting ability is desired, the total number of digits per word must be increased, decreasing the length of the addressing sequence, and also, in consequence, the integration time for the correlation detection. This effect can also be appreciated in the value of the signal-to-noise ratio, which is inversely proportional to n.

Another approach was investigated, which consisted in permitting the system to take more message digits, waiting several frames (sample times) of information, before encoding them. Noting that in the formula for γ, the value of M is equal to (1250 times the No. of frames)/n, in Table V is shown the variation of M for different lengths of blocks of information digits when encoding achieves one error correction per word.

In Fig. 20 is shown the error rate of the system for 1, 3 and 6 frames of delay. Again a slight advantage only is seen for the smaller values of probability of error. The system is increasingly degraded as the number of simultaneous transmissions increases. In this case, despite the increase of M, the performance of the system is not improved because with the increase of the block length the error rate at the output of the error-correcting decoding stage is increased.

From what has just been said, it can be concluded that the needs of the required signal addressing in this type of

TABLE IV

VARIATION OF M WITH FRAME-BY-FRAME ENCODING AND DIFFERENT VALUES OF ERROR-CORRECTING ABILITY

M is the number of digits of the pseudo-random sequence, n is the total number of digits in every 125 μs interval, and e the number of errors connected per n digits.

e	n	M
1	11	113
2	14	89
3	17	73
4	20	62
5	23	54
6	27	47

TABLE V

VARIATION OF M, FOR DIFFERENT VALUES OF INFORMATION DIGITS AND 1 ERROR-CORRECTION ABILITY

k is the number of information digits; n and M are as defined in Table IV.

k	n	M
7	11	113
14	19	131
21	26	144
28	34	147
35	41	152
42	48	156

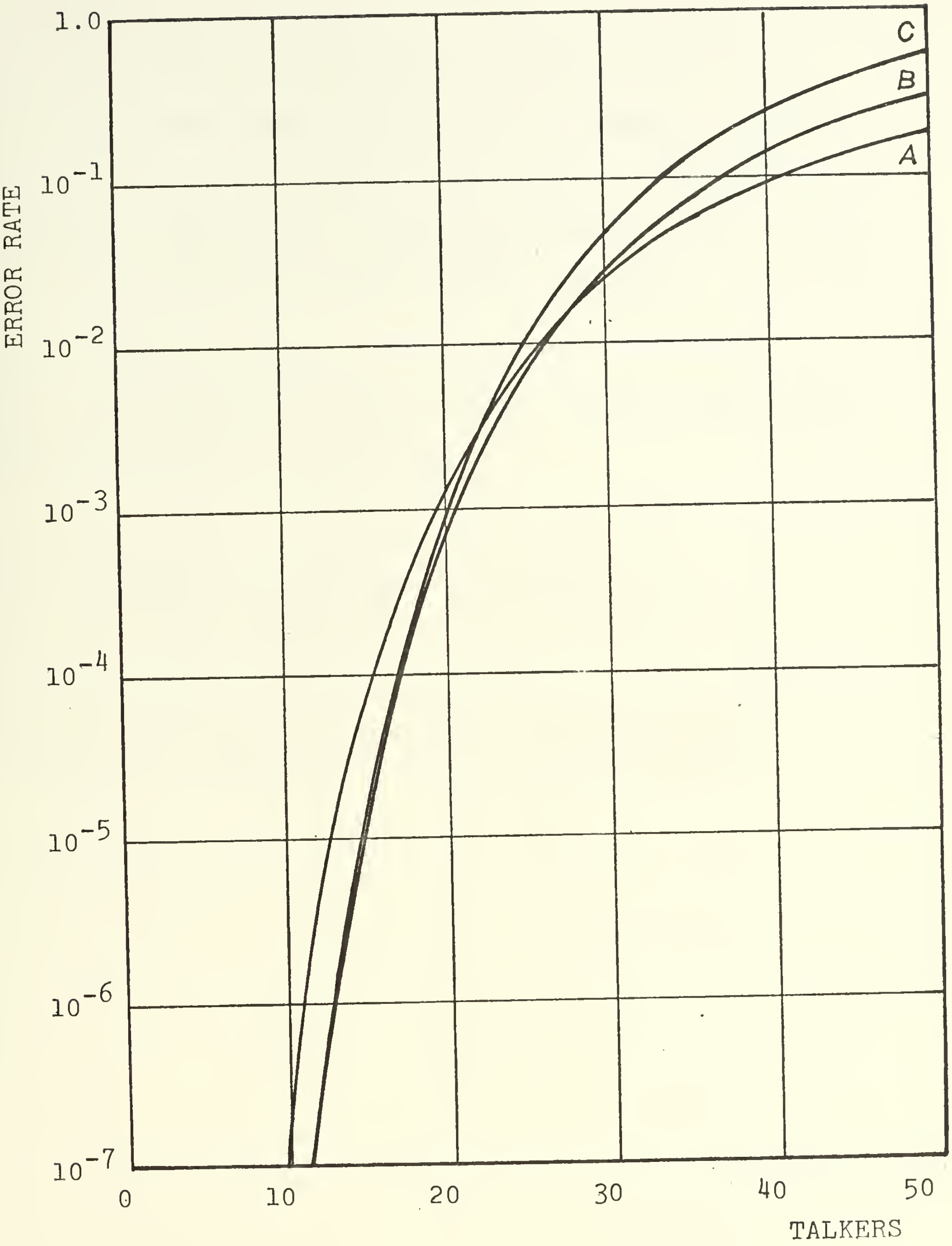

Figure 20. Probability of error for 1 error-correction and 1, 3, and 6 frames of delay. Curve A: 1 frame of delay. Curve B: 3 frames of delay. Curve C: 6 frames of delay.

RADA system for recognition of the desired signal against other simultaneous transmissions requires that integration time in the correlation detector be of adequate duration. The use of block error-correcting methods does not provide the desired improvement in the performance of the system because the addition of check digits obligates a decrease in the integration time. Only for very low values of β is the coded system slightly better than the PCM system without error-correcting ability.

VIII. CONVOLUTIONAL ENCODING OF INFORMATION

In the previous Section the effect of block encoding the PCM information pulses was investigated. It was concluded that the addition of parity check digits does not give a remarkable increase in the allowable number of simultaneous transmissions because the correlation process is deteriorated in detecting the individual pulses, increasing the channel error.

Another idea was also investigated with respect to block codes, namely the coding of blocks of several groups of PCM pulses. This increases the correlation time of the detector, achieving a slight improvement over frame-by-frame coding. This idea of increasing the correlation time can be exploited in more depth, using a different approach to the coding problem. In this Section the use of convolutional codes in RADA systems is described.

Convolutional codes are a type of parity-check code, where k information digits are mapped into m + k channel digits, by the addition of m parity-check digits. In the determination of the m check digits there not only enters into consideration the k information-digit components of the most recent block, but also a number of previous information digits. The question of how many sets a particular information digit is useful for forming check digits depends on the particular code, and this characteristic is called

the constraint length. One of the definitions given for constraint length is the number of channel digits that come out of the encoder between when a given information digit enters the encoder and when it no longer affects the encoder [Ref. 21]. Obviously, as the time that a digit stays in the encoder is increased, its presence will be reflected in more sets of m check digits and a better coding system may be obtained

Redundancy of a code can be defined as

$$\text{Redundancy} = \frac{m}{m + k} \tag{8.1}$$

and, in consequence, rate of transmission, R, becomes:

$$R = 1 - \text{Redundancy} = \frac{k}{m + k} . \tag{8.2}$$

It should be noticed that redundancy and constraint length are not directly related. This fact is of special importance in RADA systems.

The value of redundancy will affect the correlation time in the detection process explained in Section IV. The value of constraint length will not affect this parameter, and only will affect the time delay in encoding and decoding the information digits.

For decoding the convolutional codes no unique method or definite algorithm exists. One approach easily implemented is threshold decoding, which is efficient for correcting errors of short constraint lengths. Another method is sequential or probabilistic decoding.

Gallager [Ref. 21] has given an upper bound to the probability of error for sequential decoding. Based on this reference the investigation of the performance of the RADA system follows.

The general formula given by Gallager, as adapted to the notation of this thesis, is:

$$P(\text{error}) \leq 2^{k+1}\left[\frac{N}{(m+k)(1-z^2)} + \frac{1+z}{(1-z)^3}\right] e^{-NE_o(1,Q)} \tag{8.3}$$

and $$z = 2^k \cdot e^{-(m+k)E_o(1,Q)}, \quad \text{where}$$

N = constraint length of the code

m = number of check digits for every k information digits

$E_o(1,Q)$ = sequential decoding exponent

Q = input probability for the channel input

For the binary symmetric channel (BSC), $Q(o) = Q(1) = \frac{1}{2}$. This bound is valid only for a rate less than the sequential decoding exponent, or $R < E_o(1,Q)$, R being expressed in natural units (nats) per channel digits: $R = \frac{k}{m+k} \ln 2$.

Using these consideration, after rearrangement, the formulas for the upper bound for convolutional codes, applied to the BSC are:

$$P(\text{error}) \leq 2^{k+1} 2^{-N}\left[\frac{N}{(m+k)(1-z)^2} + \frac{1+z}{(1-z)^3}\right](1+2\sqrt{\beta(1-\beta)})^N$$

$$z = 2^{-m}\,(1 + 2\sqrt{\beta(1-\beta)})^{(m+k)} \;. \tag{8.4}$$

Replacing the sequential decoding exponent for its corresponding value in the BSC, the condition of validity of these formulas may be expressed as a bound in the channel error probability, as follows:

$$\beta < 0.5 - 0.5\sqrt{1 - \left(2^{\frac{m}{m+k}} - 1\right)^2} \qquad (8.5)$$

The scheme used in the detection process consists in comparing the received sequence with the most probable sequence that could be sent. The decoder compares the closeness of fit between the hypothesized transmitted sequence and the actual received sequence. Closeness of fit is expected to increase as the lengths of the respective sequences increase. If this does not occur the decoder returns to a new hypothesis and repeats the process.

One example of the application of these results follows: Suppose a 2/3 rate code; this means that in the encoding process, one check digit will be inserted for every 2 information digits. Using these values, the expression for the probability of error will be:

$$P(\text{error}) \leq 8 \cdot 2^{-N} \left(1 + 2\sqrt{\beta(1-\beta)}\right)^N \left[\frac{N}{3(1-z)^2} + \frac{1+z}{(1-z)^3}\right] \qquad (8.6)$$

$$z = 0.5 \left(1 + 2\sqrt{\beta(1-\beta)}\right)^3 .$$

The value of β, channel error probability, is determined by the earlier expression for the detection of the address in the correlation detector. Using the bipolar type of detection, β becomes:

$$\beta = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

$$\text{where} \quad \gamma = M \sqrt{\frac{3}{3L^2+L(2M-3)}} . \qquad (8.7)$$

The value of M is calculated using the average of the total number of digits sent every 125 μs. As considered before, the information source will put into the encoder 7 digits every 125 μs. At its output will emerge:

$$\frac{7}{\text{Rate}} = \frac{7(m+k)}{K} = \frac{7 \times 3}{2} = 10.5 \text{ digits}$$

With this value, $M = \frac{1250}{10.5}$, for a 10 MHz bandwidth system.

In Fig. 21 are shown L versus the error rate for constraint lengths of 40 and 60 digits, respectively. Important characteristics can be appreciated; the concavity of the curve, that is opposite to all the other cases seen till now, and the abrupt rise of the error rate near 30 talkers. For L less than 30 the system can have a very small probability of error, with relatively short constraint length.

One property of this coding method, fundamental for the application to the RADA system, is that for a given rate of transmission, the probability of error can be as low as desired, at the expense of delay time in decoding, but without the need for increasing the number of check digits.

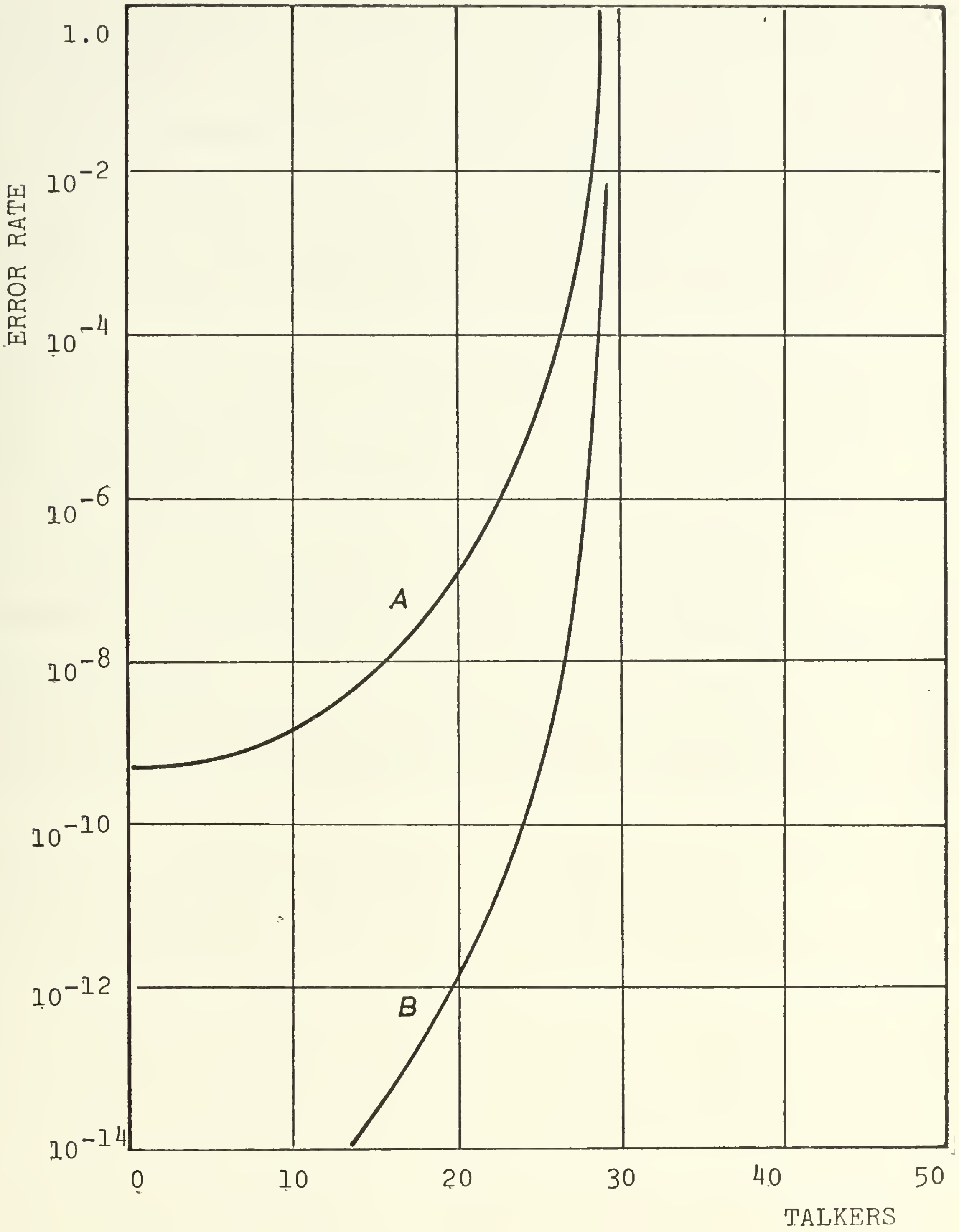

Figure 21. Probability of error for 2/3 rate convolutional code system with 40 and 60 digits of constraint length. Curve A: 40 digits of constraint length. Curve B: 60 digits of constraint length.

Thus the integration time in the correlation detector is not affected.

It is interesting to recall that this analysis for convolutional coding is based on the upper bound for the error probability. For a definite encoding and decoding process one would expect to have better performance than that described. More elaborated treatments have given calculations of the probability of error in convolutional codes, considering rates approaching the channel capacity [Refs. 22, 23]. Definite encoding and decoding processes, along with tighter bounds on the probability of error seem appropriate and worthwhile to explore in this application to RADA systems.

IX. CONCLUSIONS AND RECOMMENDATION FOR THE FUTURE

The RADA system studied in this thesis has the special characteristics that all the transmissions use only one common frequency as carrier for all the talkers and also that no time synchronization of the whole net is required. This scheme then requires that a pair of talkers must find means for recognizing their respective transmissions. The use of pseudo-random sequences as a characteristic address for each talker was the form selected as a basis of recognition. The necessity of a first detection stage for identifying the address sequence using correlation methods, before decoding the information, has imposed constraints on the system. The correlation detection of the desired address permits the rejection of the unwanted transmissions, and the longer the time taken for the correlation process the better will be the performance obtained of the system.

The principal digital method of transmitting information was by PCM. Several error-correcting encoding methods for sending the PCM binary sequences were studied on a comparative basis, in order to indicate how system performance might be thereby improved. The performance of an M-ary system was also investigated.

The results of the comparative study of encoding methods may be summarized as follows. The M-ary system makes the best use of the correlation time in the detection process

and a low channel error probability is obtained. Block codes do not provide the desired results, because the necessary addition of check digits in effect reduces the correlation time, deteriorating the detection process. Due to this fact the final performance is not significantly better than for the PCM system without any error-correcting ability.

Of all the systems studied the most promising seems to involve the use of convolutional codes. Despite the fact that in this treatment their performance was investigated using only upper bounds for the probability of error, the results are better than those for the other systems considered. Definite encoding and decoding schemes, for applying convolutional coding to RADA systems, is a promising research area.

Overall "systems" aspects of the RADA system developed in this tyesis have not been considered. This would be a study in itself. However, it should be pointed out that the bounds obtained on L, the number of talkers, are not unreasonably small. For example if L = 30, then L + 1 or 31 pairs of subscribers could be using the net simultaneously. If the use factor for the system were 3% then the total number of subscribers could be of the order of 2000.

COMPUTER PROGRAMS

SIMULATION OF THE COMMUNICATION SYSTEM USING AS NOISE THE SUM OF PSEUDO-RANDOM SEQUENCES

```
      DIMENSION X(127),Y(127)
      INTEGER IBIT(127),JBIT(127)
      READ(5,100)(IBIT(I),I=1,80)
      READ(5,101)(IBIT(I+80),I=1,47)
      READ(5,100)(JBIT(I),I=1,80)
      READ(5,101)(JBIT(I+80),I=1,47)
  100 FORMAT(80I1)
  101 FORMAT(47I1)
      DO 3 I=1,127
      IF(IBIT(I).EQ.1)GO TO 7
      X(I)=+1.
      GO TO 4
    7 X(I)=-1.
    4 IF(JBIT(I).EQ.1)GO TO 8
      Y(I)=+1.
      GO TO 3
    8 Y(I)=-1.
    3 CONTINUE
      DO 5 I=1,10
      READ(5,50)NS
   50 FORMAT(I3)
      CALL ERROR(X,Y,.5,NS, 500,ERR,SSQ,SMEAN)
      WRITE(6,60)NS,ERR,SSQ,SMEAN
   60 FORMAT(I10,3E20.8)
    5 CONTINUE
      STOP
      END


      SUBROUTINE ERROR(XX,YY,SIGN,NS,ITER,ERR,SSQ,SMEAN)
      DIMENSION XX(1),YY(1),CO(127),IX(127),IBIN(50)
      DX=.1
      VMIN=-1.05
      SS=0.
      E=0.
      SQ=0.
      SMEAN=0.
      IV=34127
      DO 6  I=1,NS
      CALL GAUSS(IV,.07,1.,W)
      CO(I)=W
    6 CONTINUE
      NBIN=21
      DO 300 I=1,NBIN
  300 IBIN(I)=0
      SQNOI=0.
      DO 8 JJ=1,ITER
      ICOUNT=1
      SUM=0.
      DO 71 I=2,NS
      CALL RANDU (IV,IW,W)
      IV=IW
      IX(I)=127.*W
   71 CONTINUE
      DO 3 I=1,127
      CALL RNOISE(YY,CO,NS,ICOUNT,IX,V)
      Y=V*XX(I)
      SUM=SUM+Y
      ICOUNT=ICOUNT+1
    3 CONTINUE
      SUM=SUM/127.
      NB=(SUM-VMIN)/DX+1.
      IF(NB.LT.1) GO TO 21
      IF(NB.GT. 21) GO TO 21
      IBIN(NB)=IBIN(NB)+1
   21 SQNOI=SUM**2+SQNOI
      SMEAN= SUM+SMEAN
```

```
      IF (SUM.LT.SIGN) GO TO 5
      E=E+1.
      GO TO 8
    5 SS=SS+1.
    8 CONTINUE
      ERR=E/(SS+E)
      AITER=ITER
      SSQ=SQNOI/AITER
      SMEAN=SMEAN/AITER
      DO 29 I=1,21
   29 WRITE(6,35) IBIN(I)
   35 FORMAT(I20)
      RETURN
      END

      SUBROUTINE PNOISE(Y,CO,NS,ICOUNT,IX,V)
      DIMENSION Y(1),CO(1),IX(1)
      V=0.
      J=ICOUNT
      DO 5 K=1,NS
    3 IF(J.LE.127) GO TO 4
      J=J-126
      GO TO 3
    4 V=V+Y(J)*CO(K)
      J=J+IX(K+1)
    5 CONTINUE
      RETURN
      END
```

SIMULATION OF THE COMMUNICATION SYSTEM USING GAUSSIAN DISTRIBUTED NOISE

```
      INTEGER IBIT(127)
      DIMENSION X(127)
      READ(5,100)(IBIT(I),I=1,80)
  100 FORMAT(80I1)
      READ(5,101)(IBIT(I+80),I=1,47)
  101 FORMAT(47I1)
      DO 3 I=1,127
      IF(IBIT(I).EQ.1)GO TO 7
      X(I)=-1.
      GO TO 3
    7 X(I)=1.
    3 CONTINUE
      DO 5 I=1,8
      READ(5,50)S
   50 FORMAT(E10.1)
      CALL GAUSER(X,.0,S,0., 500,127,ERR,SSQ)
      WRITE(6,60)S,ERR,SSQ
   60 FORMAT(3E25.5)
    5 CONTINUE
      STOP
      END


      SUBROUTINE GAUSER(XX,SIGN,S,AM,ITER,INTEG,ERR,SSQ)
      DIMENSION XX(1)
      IX=123
      SS=0.
      E=0.
      SQ=0.
      AINTEG=INTEG
      SQNOI=0.
      DO 8 JJ=1,ITER
      SUM=0.
      DO 3 I=1,INTEG
      CALL GAUSS(IX,S,AM,V)
      Y=(XX(I)+V)*XX(I)
      SUM=SUM+Y
    3 CONTINUE
      SUM=SUM/AINTEG
      SQNOI=SUM**2+SQNOI
      IF(SUM.GT.SIGN)GO TO 5
      E=E+1.
      GO TO 8
    5 SS=SS+1.
    8 CONTINUE
      ERR=E/(SS+E)
      AITER=ITER
      SSQ=SQNOI/AITER
      RETURN
      END


      SUBROUTINE GAUSS(IX,S,AM,V)
      A=0.0
      DO 50 I=1,12
      CALL RANDU(IX,IY,Y)
      IX=IY
   50 A=A+Y
      V=(A-6.)*S+AM
      RETURN
      END


      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY)5,6,6
    5 IY=IY+2147483647+1
    6 YFL=YFL*.4656613E-9
      RETURN
      END
```

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13. ABSTRACT

This thesis describes several modulation schemes to be used in a random access discrete address (RADA) system. This RADA system uses only one common frequency as carrier, and no synchronization of the net is supposed. Recognition of the transmissions between pairs of talkers involves the use of pseudo-random sequences as individual addresses of each subscriber.

Detection of a particular pseudo-random sequence, contaminated by the noise formed by the simultaneous transmissions of other sequences, is accomplished by correlation methods.

A comparative study is presented for the transmission of the information by the RADA system. The following methods are described and analyzed: an M-ary system, where the information is represented by cyclic shifts of the pseudo-random sequence used as the recipient's address, and pulse code modulation (PCM) where the information is represented by a "1" or "0".

Finally, the effect of encoding the information digits via error-correcting codes is investigated. Here, use of cyclic block codes and convolutional codes are presented. It is concluded that the use of convolutional codes would improve the performance of RADA systems.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
MULTIPLE ACCESS						
SATELLITE COMMUNICATIONS						
RANDOM ACCESS DISCRETE ADDRESS						
PSEUDO-NOISE COMMUNICATION						
PSEUDO-RANDOM SEQUENCES						
DIGITAL COMMUNICATION						
ERROR-CORRECTING CODES						

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